

AP CALCULUS
EXPONENTIAL FUNCTIONS AND THEIR DERIVATIVES

1. Use your TI-89 to check your answers.

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7. $\lim_{x \rightarrow \infty} (1.1)^x = +\infty$

8. With limits of this type, rewrite without negative exponents and remember that $\frac{1}{e^{ax}} \rightarrow 0$ as x gets huge.

$$\lim_{x \rightarrow \infty} \frac{e^{3x} - e^{-3x}}{e^{3x} + e^{-3x}} = \lim_{x \rightarrow \infty} \frac{\frac{e^{3x} - 1}{e^{3x}}}{\frac{e^{3x} + 1}{e^{3x}}} = 1$$

$$9. \lim_{x \rightarrow -\infty} \frac{e^{3x} - e^{-3x}}{e^{3x} + e^{-3x}} = \lim_{x \rightarrow -\infty} \frac{\frac{e^{3x} - 1}{e^{3x}}}{\frac{e^{3x} + 1}{e^{3x}}} = -1$$

$$10. \lim_{x \rightarrow (\pi/2)^-} e^{\frac{2}{x-1}} = e^{\frac{4}{\pi-2}} \quad (\text{In this case, if you "plug-in" the number, you get a number out and you're finished!})$$

$$11. f'(x) = \left(e^{\sqrt{x}} \right) \left(\frac{1}{2\sqrt{x}} \right) = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$$

$$\begin{aligned} 12. \frac{dy}{dx} &= (x)(2e^{2x}) + e^{2x} \\ &= e^{2x}(2x+1) \end{aligned}$$

$$\begin{aligned} 13. h'(t) &= \frac{1}{2} (1-e^t)^{-1/2} (-e^t) \\ &= -\frac{e^t}{2\sqrt{1-e^t}} \end{aligned}$$

$$\begin{aligned} 14. \frac{dy}{dx} &= e^{x \cos x}(-x \sin x + \cos x) \\ &= e^{x \cos x}(\cos x - x \sin x) \end{aligned}$$

$$\begin{aligned} 15. g(x) &= e^{-x^{-1}} \\ g'(x) &= e^{-x^{-1}}(x^{-2}) \\ &= \frac{e^{-1/x}}{x^2} \end{aligned}$$

$$16. \frac{dy}{dx} = [\sec^2(e^{3x-2})] (e^{3x-2}) (3)$$

$$= 3e^{3x-2} \sec^2(e^{3x-2})$$

$$17. f'(x) = \frac{(1+e^x)(3e^{3x}) - (e^{3x})(e^x)}{(1+e^x)^2}$$

$$= \frac{e^{3x}(3+2e^x)}{(1+e^x)^2}$$

$$18. \frac{dy}{dx} = ex^{e-1} \text{ (} e \text{ is just a constant-standard power rule applies.)}$$

19. First, get rid of negative exponents and get a simplified fraction.

$$y = \frac{e^{2x} + 1}{e^{2x} - 1}$$

$$\frac{dy}{dx} = \frac{(e^{2x}-1)(2e^{2x}) - (e^{2x}+1)(2e^{2x})}{(e^{2x}-1)^2}$$

$$= -\frac{4e^{2x}}{(e^{2x}-1)^2}$$

$$20. f'(x) = [\sec(e^{\tan x^2}) \tan(e^{\tan x^2})] (e^{\tan x^2}) (\sec^2 x^2)(2x)$$

21. Point

Given as $(\pi, 0)$.

Slope

$$\frac{dy}{dx} = (e^{-x}) (\cos x) - e^{-x} \sin x \longrightarrow \left. \frac{dy}{dx} \right|_{x=\pi} = -e^{-\pi}$$

Equation of tangent

$$y - 0 = -e^{-\pi}(x - \pi)$$

22. Point

$$\text{Given as } \left(1, \frac{1}{e}\right)$$

Slope

$$f'(x) = (x^2)(-e^{-x}) + (e^{-x})(2x) \longrightarrow f'(1) = \frac{1}{e}$$

Equation of tangent

$$y - \frac{1}{e} = \frac{1}{e}(x - 1)$$

$$23. \quad [-\sin(x-y)] \left(1 - \frac{dy}{dx}\right) = xe^x + e^x$$

$$-\sin(x-y) + \frac{dy}{dx} \sin(x-y) = xe^x + e^x$$

$$\frac{dy}{dx} = \frac{xe^x + e^x + \sin(x-y)}{\sin(x-y)}$$

24. Point

Given as $(0, 2)$

Slope

$$2e^{xy} \left(x \frac{dy}{dx} + y \right) = 1 + \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1 - 2ye^{xy}}{2xe^{xy} - 1}$$

$$\left. \frac{dy}{dx} \right|_{(0,2)} = 3$$

Equation of tangent

$$y - 2 = 3(x - 0)$$