1. $f'(x) = \dfrac{1}{x+1}$

2. $$\begin{aligned} f'(x) &= x^2\left(\frac{-2x}{1-x^2}\right) + 2x\ \ln(1-x^2) \\ &= 2x\ln(1-x^2) - \frac{2x^3}{1-x^2} \end{aligned}$$

3. $$f'(x) = \frac{2x}{(x^2-4)\ln 3}$$

or

$$f'(x) = \frac{(\log_3 e)\ 2x}{x^2-4}$$

4. $$\begin{aligned} f'(x) &= x\,\frac{1}{x} + (\ln x)(1) \\ &= 1 + \ln x \\ f''(x) &= \frac{1}{x} \end{aligned}$$

5. $$\begin{aligned} g'(x) &= \frac{\log_{10} e}{x} \\ g''(x) &= -\frac{\log_{10} e}{x^2} \end{aligned}$$

or

$$\begin{aligned} g'(x) &= \frac{1}{x\ln 10} \\ g''(x) &= -\frac{1}{x^2\ln 10} \end{aligned}$$

6. $$\begin{aligned} f'(x) &= \frac{\sqrt{x}}{x} + \frac{\ln x}{2\sqrt{x}} \\ &= \frac{2x + x\ln x}{2x\sqrt{x}} \\ &= \frac{2+\ln x}{2\sqrt{x}} \end{aligned}$$

7. $$\begin{aligned} g(x) &= \ln(a-x) - \ln(a+x) \\ g'(x) &= -\frac{1}{a-x} - \frac{1}{a+x} \\ &= \frac{1}{x-a} - \frac{1}{x+a} \\ &= \frac{2a}{x^2-a^2} \end{aligned}$$

8. $$g(x) = \frac{1}{2}\ \ln x$$

$$g'(x) = \frac{1}{2x}$$

9. $$f'(t) = \frac{4t^3 - 2t}{(t^4 - t^2 + 1)\ln 2}$$

or

$$f'(t) = \frac{(4t^3 - 2t)\log_2 e}{t^4 - t^2 + 1}$$

10. $$g'(u) = \frac{(1 + \ln u)\left(-\frac{1}{u}\right) - (1 - \ln u)\left(\frac{1}{u}\right)}{(1 + \ln u)^2}$$

$$= -\frac{2}{u(1 + \ln u)^2}$$

11. $$\frac{dy}{dx} = \left[3\ln^2(\sin x)\right]\ \frac{\cos x}{\sin x}$$

$$= (3\cot x)\ln^2(\sin x)$$

12. $$\frac{dy}{dx} = \frac{(1 + x^2)\left(\frac{1}{x}\right) - 2x\ln x}{(1 + x^2)^2}$$

$$= \frac{(1 + x^2) - 2x^2\ln x}{x(1 + x^2)^2}$$

13. $$F'(x) = e^x\left(\frac{1}{x}\right) + (\ln x)(e^x)$$

$$= \frac{e^x}{x} + e^x\ln x$$

$$= e^x\left(\frac{1}{x} + \ln x\right)$$

14. $$f'(t) = (\pi^{-t})(\ln \pi)(-1)$$

Note: f is of the form $a^{f(x)}$

15. $$h'(t) = 3t^2 - 3^t(\ln 3)(1)$$

$$= 3t^2 - 3^t\ln 3$$

16. $$\frac{dy}{dx} = \frac{-e^{-x} + xe^{-x}(-1) + e^{-x}}{e^{-x} + xe^{-x}}$$

$$= -\frac{x}{1 + x}$$

17.
$$\ln y = \ln x^{\sin x}$$
$$\ln y = \sin x \ln x$$
$$\frac{1}{y}\,\frac{dy}{dx} = (\sin x)\left(\frac{1}{x}\right) + (\ln x)(\cos x)$$
$$\frac{dy}{dx} = x^{\sin x}\left[\frac{\sin x}{x} + \ln x \cos x\right]$$

18.
$$\ln y = \ln x^{e^x}$$
$$\ln y = e^x \ln x$$
$$\frac{1}{y}\,\frac{dy}{dx} = e^x\left(\frac{1}{x}\right) + (\ln x)(e^x)$$
$$\frac{dy}{dx} = x^{e^x}\left(\frac{e^x}{x} + e^x \ln x\right)$$

19. Point

Given as $(e, 0)$.

Slope

$$\frac{dy}{dx} = \frac{\frac{1}{x}}{\ln x} = \frac{1}{x \ln x} \longrightarrow \left.\frac{dy}{dx}\right|_{(e,0)} = \frac{1}{e}$$

Equation of tangent

$$y - 0 = \frac{1}{e}(x - e)$$

20. Point

Given as $(1, 10)$.

Slope

$$\frac{dy}{dx} = 10^x \ln 10 \longrightarrow \left.\frac{dy}{dx}\right|_{(1,10)} = 10 \ln 10$$

Equation of tangent

$$y - 10 = 10 \ln 10(x - 1)$$

21. $2c + \ln c = 2 \longrightarrow c = 1 \longrightarrow (1, 2)$ is on f.

Since $(1, 2)$ is on f, $\left(f^{-1}\right)'(2) = \frac{1}{f'(1)}$

$f'(x) = 2 + \frac{1}{x} \longrightarrow f'(1) = 3 \longrightarrow \left(f^{-1}\right)'(2) = \frac{1}{3}$.