1. $V = x^3$

$$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$$

2. $xy = 1$

$$x \frac{dy}{dt} + y \frac{dx}{dt} = 0$$

For $x = 2$ and $\frac{dx}{dt} = 4$,

$$2 \frac{dy}{dt} + \frac{1}{2}(4) = 0$$

$$\frac{dy}{dt} = -1$$

3. Find $\frac{dr}{dt}$ when $r = 5$.

$$V = \frac{4}{3}\pi r^3$$

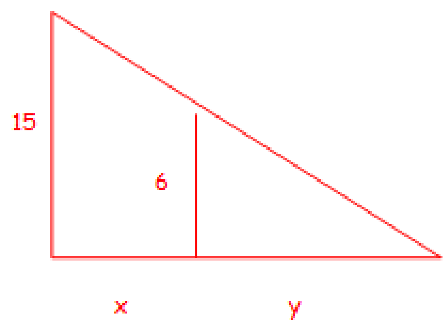
$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

For $\frac{dV}{dt} = -1$ and $r = 5$,

$\frac{dr}{dt} = -\frac{1}{100\pi} \longrightarrow \frac{dd}{dt} = -\frac{2}{100\pi}$ where $\frac{dd}{dt}$ is the rate of change in the diameter.

Therefore, the diameter is decreasing at $\frac{1}{50\pi}$ cm/min.

4. Diagram.



<u>Shadow length</u>

Find $\dfrac{dy}{dt}$ when $x = 40$ and $\dfrac{dx}{dt} = 5$.

$$\frac{15}{6} = \frac{x+y}{y}$$

$$3y = 2x$$

$$3\,\frac{dy}{dt} = 2\,\frac{dx}{dt}$$

For $x = 5$,

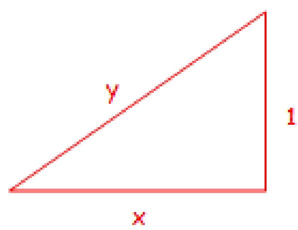
$\dfrac{dy}{dt} = \dfrac{10}{3} \longrightarrow$ the shadow in growing larger at $\dfrac{10}{3}$ ft/sec .

<u>Tip of shadow</u>

$$\frac{dx}{dt} + \frac{dy}{dt} = \frac{25}{3}$$

Therefore the tip of the shadow is moving at $\dfrac{25}{3}$ feet per second.

5. Diagram.



$$\text{If } \frac{dx}{dt} = 500 \text{ find } \frac{dy}{dt} \text{ when } x = 2.$$

$$x^2 + 1 = y^2$$

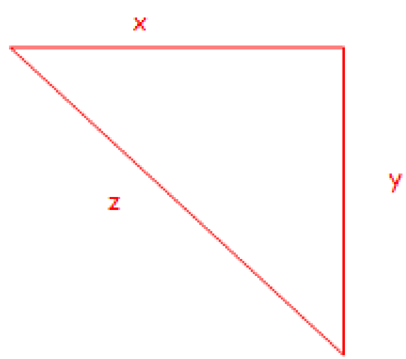
$$2x\,\frac{dx}{dt} = 2y\,\frac{dy}{dt}$$

$$\text{When } x = 2, y = \sqrt{5} \text{ and } \frac{dx}{dt} = 500,$$

$$\frac{dy}{dt} = \frac{1000}{\sqrt{5}}$$

Therefore the line of sight distance is increasing at $\frac{1000}{\sqrt{5}}$ miles per hour.

6. Diagram.



$$\text{For } \frac{dx}{dt} = 25 \text{ and } \frac{dy}{dt} = 60, \text{ find } \frac{dz}{dt} \text{ when } x = 50 \text{ and } y = 120.$$

Both rates are positive because both are getting larger.

$$z^2 = x^2 + y^2$$

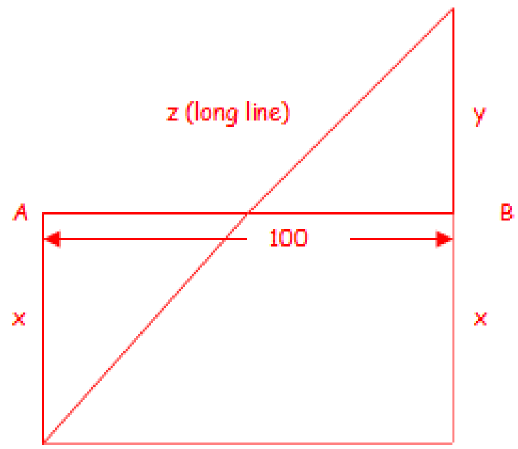
$$2z\,\frac{dz}{dt} = 2x\,\frac{dx}{dt} + 2y\,\frac{dy}{dt}$$

$$\text{For } x = 50, y = 120 \text{ and } z = 130,$$

$$\frac{dz}{st} = 65$$

Therefore the distance between the cars is increasing at 65 miles per hour.

7. Diagram.



$$\text{If } \frac{dy}{dt} = 25 \text{ and } \frac{dx}{dt} = 35, \text{ find } \frac{dz}{dt} \text{ when } x = 140 \text{ and } y = 100.$$

$$z^2 = (x+y)^2 + 100^2$$

$$2z\,\frac{dz}{dt} = 2(x+y)\left(\frac{dx}{dt} + \frac{dy}{dt}\right) = 0$$

$$\text{For } x = 140, y = 100, z = 260, \frac{dy}{dt} = 25 \text{ and } \frac{dx}{dt} = 35,$$

$$\frac{dz}{dt} = \frac{720}{13}$$

Therefore the distance between the ships is increasing at $\frac{720}{13}$ kilometers per hour.

8. $\text{If } \frac{dA}{dt} = 2 \text{ and } \frac{dh}{dt} = 1, \text{ find } \frac{db}{dt} \text{ when } h = 10 \text{ and } A = 100$

$$A = \frac{1}{2}\,b\,h$$

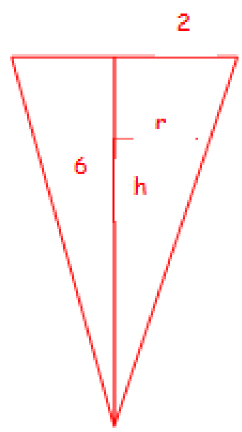
$$\frac{dA}{dt} = \left(\frac{1}{2}\,b\right)\frac{dh}{dt} + h\,\frac{1}{2}\,\frac{db}{dt}$$

$$\text{For } h = 10, A = 100\ (b = 20), \frac{dA}{dt} = 2 \text{ and } \frac{dh}{dt} = 1,$$

$$\frac{db}{dt} = -\frac{8}{5}$$

Therefore the base is decreasing at $\frac{8}{5}$ centimeters per minute.

9. Diagram.



I did the problem in meters and converted the final answer into centimeters at the very end.

Let x be the rate at which water is being pumped in.

$$\frac{dV}{dt} = x - .01$$

We need to find x when $\frac{dh}{dt} = .2$ and $h = 2$.

$$V = \frac{1}{3}\pi r^2 h$$

We know $\frac{6}{2} = \frac{h}{r} \longrightarrow r = \frac{h}{3}$.

$$V = \frac{\pi}{27} h^3$$

$$\frac{dV}{dt} = \frac{\pi}{9} h^2 \frac{dh}{dt}$$

$$x - .01 = \frac{\pi}{9}(4)(.2)$$

$$x \approx .289$$

Therefore water is being pumped in at approximately .289 cubic meters per minute. . . or 289,000 cubic centimeters per minute.

10. This is a cone problem so we need to get r in terms of h again.

Diameter=height

$$2r = h$$

$$r = \frac{h}{2}$$

Now, we need to find $\dfrac{dh}{dt}$ when $h = 10$.

$$V = \frac{1}{3}\,\pi r^2 h$$

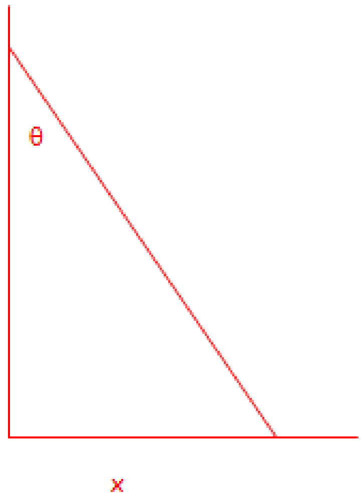
$$\frac{dV}{dt} = \frac{\pi}{4}\,h^2 \frac{dh}{dt}$$

For $\dfrac{dV}{dt} = 30$ and $h = 10$,

$$\frac{dh}{dt} = \frac{6}{5\pi}$$

Therefore the height of the pile is increasing at $\dfrac{6}{5\pi}$ feet per minute.

11. Diagram.



$$\text{Find } \frac{d\theta}{dt} \text{ when } \theta = \frac{\pi}{4} \text{ and } \frac{dx}{dt} = 2.$$

$$\sin\theta = \frac{x}{10}$$

$$\sin\theta = \frac{1}{10}\,x$$

$$\cos\theta\,\frac{d\theta}{dt} = \frac{1}{10}\,\frac{dx}{dt}$$

$$\text{For } \theta = \frac{\pi}{4} \text{ and } \frac{dx}{dt} = 2,$$

$$\frac{d\theta}{dt} = \frac{\sqrt{2}}{5}$$

Therefore the angle between the top of the ladder and the wall is increasing at $\frac{\sqrt{2}}{5}$ radians per second.