

Note: Many of these problems involve solving non-linear inequalities. You can use a sign chart or a sketch ... it does not matter because it is not technically part of your solution and does not have to be shown.

1. $v(t) = 3t^2 - 24t - 36$

Critical Numbers

- $v \geq 0 \quad \forall t \geq 0$
- $f(t) = 0$ when $t = 6$ or $t = 2$

Since $v(t) > 0$ on $(0, 2) \cup (6, \infty)$, the particle is moving to the right on $(0, 2) \cup (6, \infty)$.

Since $v(t) < 0$ on $(2, 6)$, the particle is moving to the left on $(2, 6)$.

Since $v(t) > 0$ on $(0, 2)$ and $v(t) < 0$ on $(2, 6)$ the particle changes direction at $t = 2$.

Since $v(t) < 0$ on $(2, 6)$ and $v(t) > 0$ on $(6, \infty)$, the particle changes direction at $t = 6$.

2. Time to ground

Object will hit the ground when $s(t) = 0$. $s(t) = 0$ when $-16t^2 + 638 = 0 \rightarrow t \approx 6.315$ $\therefore$ the object hits the ground after approximately 6.315 seconds.

Velocity at impact

$v(t) = -32t \rightarrow v(6.315) \approx -202.069$ $\therefore$ the velocity at impact is approximately 202.069 feet per second downward.

3. Maximum Height

At maximum height $v(t) = 0$.

$v(t) = -32t + 30 \rightarrow v(t) = 0$ when $t \approx .938$.

Now, $s(.938) \approx 14.063$ $\therefore$ the max height is about 14.063 feet.

Velocity at impact

At impact $s(t) = 0 \rightarrow t = 0$ or $t \approx 1.865$.

Since $v(1.865) \approx -30.000$ the velocity at impact is about 30.000 feet per second down.

4. Average velocity on (.5,.75)

$v_{avg} = \frac{x(t_2) - x(t_1)}{t_2 - t_1} = 12$ $\therefore$ the average velocity on (.5,.75) is 12 feet per second.

Instantaneous velocity

$v(t) = -32t + 32$

$v(.5) = 16 \rightarrow$ at $t = .5$, the velocity is 16 feet per second .

$v(.75) = 8 \rightarrow$ at $t = .75$, the velocity is 8 feet per second .

Speed

Speed = $|v| \longrightarrow$ speed at $t = .5$ is 16 feet per second and speed at $t = .75$ is 8 feet per second.

Time to maximum height

At maximum height, $v(t) = 0 \longrightarrow v(t) = 0$ when $t = 1 \therefore$ stone reaches max height at 1 second.

Maximum Height

$x(1) = 16 \longrightarrow$ maximum height is 16 feet.

Time to ground

At impact, $x(t) = 0 \longrightarrow x(t) = 0$ when $t = 0$ or $t = 2$ the stone reaches the ground at $t = 2$ seconds.

Velocity at impact

$v(2) = -32 \longrightarrow$ the velocity at impact is 32 feet per second downward.

5. We need to know when the ball hits the cushion, then we can find the velocity.

The ball hits the cushion when $s(t) = 39 \longrightarrow 100t^2 + 100t = 39 \longrightarrow t = .300$ or $t = -1.300$. We choose $t = .300$.

$v(t) = 200t + 100 \longrightarrow v(.300) = 160 \longrightarrow$ the velocity when it hits the cushion is 160 cm/sec.

6. $s(t) = -16t^2 + 96t + 112 \longrightarrow s(t) = 0$ when $t = -1$ or $t = 7$.

$v(t) = -32t + 96 \longrightarrow v(t) = 0$ when $t = 3$.

Time to maximum height

At max height, $v(t) = 0 \longrightarrow t = 3$ therefore the ball reaches its max height at 3 seconds.

Maximum height

Maximum height is $s(3) = 256$ therefore the maximum height is 256 feet.

Time to impact

At impact $s(t) = 0 \longrightarrow t = 7$ therefore the ball hits the ground 7 seconds after it is thrown.

Velocity at impact

$v(7) = -128$ therefore the velocity of the ball when it hits the ground is 128 feet per second downward.