1. $2x + 3 + \left(x\,\frac{dy}{dx} + y\right) = 0$

$$\frac{dy}{dx} = \frac{-2x - 3 - y}{x}$$

2. $4y\,\frac{dy}{dx} + x\,\frac{dy}{dx} + y = 2x$

$$\frac{dy}{dx} = \frac{2x - y}{4y + x}$$

3. $2x - \left(x\,\frac{dy}{dx} + y\right) + 3y^2\,\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = \frac{y - 2x}{3y^2 - x}$$

4. $4y\,\frac{dy}{dx} + \frac{1}{3}(xy)^{-2/3}\left(x\,\frac{dy}{dx} + y\right) = 6x$

$$4y\,\frac{dy}{dx} + \frac{1}{3\sqrt[3]{x^2y^2}}\left(x\,\frac{dy}{dx} + y\right) = 6x$$

$$4y\,\frac{dy}{dx} + \frac{x}{3\sqrt[3]{x^2y^2}}\,\frac{dy}{dx} + \frac{y}{3\sqrt[3]{x^2y^2}} = 6x$$

$$\frac{dy}{dx} = \frac{6x - \dfrac{y}{3\sqrt[3]{x^2y^2}}}{4y + \dfrac{x}{3\sqrt[3]{x^2y^2}}}$$

$$\frac{dy}{dx} = \frac{18x\sqrt[3]{x^2y^2} - y}{12y\sqrt[3]{x^2y^2} + x}$$

5. $4x^3 + 4y^3\,\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = -\frac{x^3}{y^3}$$

6. $\dfrac{(x-y)\,\dfrac{dy}{dx} - y\left(1 - \dfrac{dy}{dx}\right)}{(x-y)^2} = 2x$

$$\frac{x\,\dfrac{dy}{dx} - y\,\dfrac{dy}{dx} - y + y\,\dfrac{dy}{dx}}{(x-y)^2} = 2x$$

$$x\,\frac{dy}{dx} - y = 2x(x-y)^2$$

$$\frac{dy}{dx} = \frac{2x(x-y)^2 + y}{x}$$

7. $(x)\left(\frac{1}{2}\right)(1+y)^{-1/2}\,\frac{dy}{dx}+(1+y)^{1/2}+(y)\left(\frac{1}{2}\right)(1+2x)^{-1/2}(2)+(1+2x)^{1/2}\,\frac{dy}{dx}=2$

$$\frac{x}{2\sqrt{1+y}}\,\frac{dy}{dx}+\sqrt{1+y}+\frac{y}{\sqrt{1+2x}}+\sqrt{1+2x}\,\frac{dy}{dx}=2$$

$$\frac{dy}{dx}=\frac{2-\sqrt{1-y}-\dfrac{y}{\sqrt{1+2x}}}{\dfrac{x}{2\sqrt{1+y}}+\sqrt{1+2x}}$$

8. $[-\sin(x-y)]\left(1-\frac{dy}{dx}\right)=y\cos x+(\sin x)\left(\frac{dy}{dx}\right)$

$$\frac{dy}{dx}\sin(x-y)-\sin(x-y)=y\cos x+\frac{dy}{dx}\sin x$$

$$\frac{dy}{dx}=\frac{y\cos x+\sin(x-y)}{\sin(x-y)-\sin x}$$

9. $x\,\frac{dy}{dx}+y=\left[-\csc^2(xy)\right]\left[x\,\frac{dy}{dx}+y\right]$

$$x\,\frac{dy}{dx}+y=-x\,\frac{dy}{dx}\csc^2(xy)-y\csc^2(xy)$$

$$x\,\frac{dy}{dx}+x\,\frac{dy}{dx}\csc^2(xy)=-y-y\csc^2(xy)$$

$$\frac{dy}{dx}=\frac{-y-y\csc^2(xy)}{x+x\csc^2(xy)}$$

10. $4y^3\,\frac{dy}{dx}+2x^2y\,\frac{dy}{dx}+2xy^2+x^4\,\frac{dy}{dx}+4x^3y=\frac{dy}{dx}$

$$\frac{dy}{dx}=\frac{-2xy^2-4x^3y}{4y^3+2x^2y+x^4-1}$$

11. $(x)(3)[f(x)^2]f'(x)+[f(x)^3]+xf'(x)+f(x)=0$

Now, $x=3$ and $f(3)=1$

$$(3)(3)[f(3)^2]f'(3)+[f(3)^3]+3f'(3)+f(3)=0$$

Solving for $f'(3)\ldots$

$$f'(3)=-\frac{1}{6}$$

12. $2\,g(x)\,g'(x)+12=x^2\,g'(x)+2x\,g(x)$

Now, $x=4$ and $g(4)=12$

$$2\,g(4)\,g'(4)+12=16\,g'(4)+8\,g'(x)$$

$$g'(4)=\frac{21}{2}$$

13. <u>Slope</u>

$$\frac{1}{16}x^2 - \frac{1}{9}y^2 = 1$$

$$\frac{1}{8}x - \frac{2}{9}y\,\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{9x}{16y}$$

$$\left.\frac{dy}{dx}\right|_{(-5,9/4)} = -\frac{5}{4} \longrightarrow m_T = -\frac{5}{4}$$

<u>Point</u>

Given as $\left(-5, \frac{9}{4}\right)$

<u>Equation of tangent</u>

$y - \frac{9}{4} = -\frac{5}{4}(x+5)$

14. <u>Slope</u>

$$y^2 = 2x^3 - x^4$$

$$2y\,\frac{dy}{dx} = 6x^2 - 4x^3$$

$$\frac{dy}{dx} = \frac{6x^2 - 4x^3}{2y}$$

$$\left.\frac{dy}{dx}\right|_{(1,1)} = 1 \longrightarrow m_T = 1$$

<u>Point</u>

Given as $(1, 1)$

<u>Equation of tangent</u>

$y - 1 = 1(x - 1)$

15. Slope

$$4(x^2+y^2)\left(2x+2y\,\frac{dy}{dx}\right)=25\left(2x-2y\,\frac{dy}{dx}\right)$$

$$\frac{dy}{dx}=\frac{25x-4x(x^2+y^2)}{4y(x^2+y^2)+25y}$$

$$\left.\frac{dy}{dx}\right|_{(3,1)}=-\frac{9}{13}\ \longrightarrow\ m_T=-\frac{9}{13}$$

Point

Given as $(3,1)$

Equation of tangent

$y-1=-\frac{9}{13}(x-3)$

16. Intersections

$$2x^2+y^2=3 \text{ and } x=y^2 \text{ therefore}$$

$$2y^4+y^2=3$$

$$y=1 \text{ or } y=-1 \ \longrightarrow\ \text{ the points are } (1,1) \text{ and } (1,-1).$$

Slope of tangent to $2x^2+y^2=3$

$\frac{dy}{dx}=-\frac{2x}{y}$

Slope of tangent to $x=y^2$

$\frac{dy}{dx}=\frac{1}{2y}$

Slopes at $(1,1)$

$-\frac{2x}{y}=-2$ and $\frac{1}{2y}=\frac{1}{2}$

Since $(-2)\left(\frac{1}{2}\right)=-1$ the tangents are perpendicular and the curves are orthogonal at $(1,1)$.

Slopes at $(1,-1)$

$-\frac{2x}{y}=2$ and $\frac{1}{2y}=-\frac{1}{2}$

Since $(2)\left(-\frac{1}{2}\right)=-1$ the tangents are perpendicular and the curves are orthogonal at $(1,-1)$.

17. Slope from derivative

$\frac{dy}{dx} = \frac{-2xy^2 - y}{2x^2y + x}$

Points–from slope = slope

$$\frac{-2xy^2 - y}{2x^2y + x} = -1$$

$$-2xy^2 - y = -2x^2y - x$$

$$2xy^2 + y = 2x^2y + x$$

$$y(2xy + 1) = x(2xy + 1)$$

$$y(2xy + 1) - x(2xy + 1) = 0$$

$$(2xy + 1)(y - x) = 0$$

$$2xy + 1 = 0 \text{ or } y = x$$

$$xy = -\frac{1}{2} \text{ or } x = y$$

Substituting these into the original curve yields…

If $xy = -\frac{1}{2} \longrightarrow \frac{1}{4} - \frac{1}{2} \neq 2 \therefore xy = -\frac{1}{2}$ yields no solutions .

If $x = y \longrightarrow x^4 + x^2 = 2 \longrightarrow x = 1$ or $x = -1$

This yields four points…from substituting into the original equation.

$$(1, 1), (1, -2), (-1, 2) \text{ and } (-1, -1)$$

But we needed $x = y \longrightarrow$ the only point that satisfies the criteria are $(1, 1)$ and $(-1, -1)$.

18. Slope of normal

$$2x - \left(x\,\frac{dy}{dx} + y\right) + 2y\,\frac{dy}{dx} = 0$$

Point

$$\frac{dy}{dx} = \frac{y - 2x}{2y - x}$$

$$\left.\frac{dy}{dx}\right|_{(-1,1)} = 1 \;\longrightarrow\; m_T = 1 \;\longrightarrow\; m_\perp = -1$$

Given as $(-1, 1)$.

Equation of normal

$y - 1 = -1(x + 1) \;\longrightarrow\; y = -x$

Intersect curve and normal

Substituting $y = -x$ into the curve yields ...

$$x^2 + x^2 + x^2 = 3$$

$$3x^2 = 3$$

$$x = 1 \text{ or } x = -1$$

This gives us the points $(1, -1)$ and $(-1, 1)$.

We already know $(-1, 1)$ is an intersection so the other point is $(1, -1)$.