1. $$\frac{1}{3\sqrt[3]{x^2}} - \frac{1}{3\sqrt[3]{y^2}}\,\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{\sqrt[3]{y^2}}{\sqrt[3]{x^2}}$$

2. $$\frac{1}{a}\,x^2 - \frac{1}{b}\,y^4 = 1$$

$$\frac{2}{a}\,x - \frac{4}{b}\,y^3\,\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{2xb}{4ay^3}$$

3. $$2x^3\,\frac{dy}{dx} + 6x^2y + (3x)\left(3y^2\,\frac{dy}{dx}\right) + 3y^3 = 0$$

$$\frac{dy}{dx} = \frac{-6x^2y - 3y^3}{2x^3 + 9xy^2}$$

4. $$1 = [\cos(x+y)]\left[1 + \frac{dy}{dx}\right]$$

$$1 = \cos(x+y) + \frac{dy}{dx}\,\cos(x+y)$$

$$\frac{dy}{dx} = \frac{1 - \cos(x+y)}{\cos(x+y)}$$

5. $$[-\sin(x+y)]\left[1 + \frac{dy}{dx}\right] = y\cos x + \sin x\,\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{-\sin(x+y) - y\cos x}{\sin(x+y) + \sin x}$$

6. $$3x^{-1} - 2y^{-1} = 2x$$

$$-3x^{-2} + 2y^{-2}\,\frac{dy}{dx} = 2$$

$$\frac{dy}{dx} = \frac{2x^2y^2 + 3y^2}{2x^2}$$

7. $$\frac{dy}{dt} = 3t^2 - 12 \longrightarrow v(t) = 3t^2 - 12$$

$$\frac{d^2y}{dt^2} = 6t \longrightarrow a(t) = 6t$$

8. $v(t) = 3t^2 - 12$

   i. $v \ \exists \ \forall \ t \in [0, \infty)$

   ii. $v(t) = 0$ when $t = 2$ or $t = -2$ but -2 is not in $[0, \infty)$

   Since $v(t) < 0$ on $[0, 2)$ the particle is moving down on $[0, 2)$

   Since $v(t) > 0$ on $(2, \infty)$ the particle is moving up on $(2, \infty)$

9. <u>If r is a constant</u>

   $V = \frac{1}{3}\pi r^2 h \longrightarrow \frac{dV}{dt} = \frac{1}{3}\pi r^2 \frac{dh}{dt}$

   <u>If h is a constant</u>

   $V = \frac{1}{3}\pi r^2 h \longrightarrow \frac{dV}{dt} = \frac{2}{3}\pi r \frac{dr}{dt} h \longrightarrow \frac{dV}{dt} = \frac{2\pi r h}{3} \frac{dr}{dt}$

10. $S = 6x^2 \longrightarrow \frac{dS}{dt} = 12x \frac{dx}{dt} \left(\text{But now we need } \frac{dx}{dt}. \right)$

    We know $V = x^3$ so $\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$.

    For $\frac{dV}{dt} = 10$ and $x = 30 \longrightarrow \frac{dx}{dt} = \frac{1}{270}$

    Now back to $\frac{dS}{dt} \ldots$

    $\frac{dS}{dt} = 12x \frac{dx}{dt}$

    $\frac{dS}{dt} = (12)(30) \frac{1}{270} \longrightarrow \frac{dS}{dt} = \frac{4}{3}$

    Therefore the surface area in increasing at $\frac{4}{3}$ square centimeters per minute.

11. We need to find $\frac{dh}{dt}$ when $h = 5$.

    $V = \frac{1}{3}\pi r^2 h$

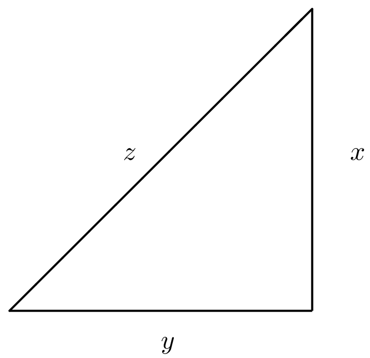
    Since $\frac{10}{h} = \frac{3}{r} \longrightarrow r = \frac{3h}{10}$

    Now, $\frac{3\pi}{100} h^3 \longrightarrow \frac{dV}{dt} = \frac{9\pi}{100} h^2 \frac{dh}{dt}$

    For $h = 5$ and $\frac{dV}{dt} = 2 \longrightarrow \frac{dh}{dt} = \frac{8}{9\pi} \approx .283$

    Therefore the height of the water is increasing at about .283 centimeters per second.

12. We need to find $\frac{dz}{dt}$ when $x = 60, y = 45$ and $z = 75$ (after 3 seconds).



$z^2 = x^2 + y^2 \longrightarrow 2z\,\frac{dz}{dt} = 2x\,\frac{dx}{dt} + 2y\,\frac{dy}{dt}$

For $x = 60, y = 45, z = 75, \frac{dy}{dt} = 15$ and $\frac{dx}{dt} = 5 \longrightarrow \frac{dz}{dt} = 13$

Therefore the distance between the boy and the balloon is increasing at 13 feet per second.

13. Linearization of f at $x = 0$

$f(0) = 1$

$f'(x) = \frac{1}{\sqrt[3]{(1+3x)^2}} \longrightarrow f'(0) = 1 \longrightarrow L(x) = 1 + x$

To find $\sqrt[3]{1.03} = \sqrt[3]{1 + 3(.01)}$ we will need to find $L(.01)$.

$L(.01) \approx 1.010 \longrightarrow \sqrt[3]{1.03} \approx 1.010$

14. $f(3) = 4$

$f'(x) = -\frac{x}{\sqrt{25 - x^2}} \longrightarrow f'(3) = -\frac{3}{4}$

Therefore $L(x) = 4 - \frac{3}{4}(x - 3)$.