

## AP CALCULUS

TANGENTS, VELOCITY AND OTHER RATES OF CHANGE

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$$\begin{aligned}
 1. \quad f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{(x+h)^2 + 2(x+h) - (x^2 + 2x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 + 2x + 2h - x^2 - 2x}{h} \\
 &= 2x + 2
 \end{aligned}$$

$$f'(-3) = -4 \longrightarrow m_T = -4$$

$$\begin{aligned}
 2. \quad \frac{dy}{dx} = f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{[1 - 2(x+h) - 3(x+h)^2] - (1 - 2x - 3x^2)}{h} \\
 &= -2 - 6x \\
 \left. \frac{dy}{dx} \right|_{x=-2} &= 10 \longrightarrow m_T = 10
 \end{aligned}$$

$$\begin{aligned}
 3. \quad \frac{dy}{dx} = f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{1}{h} \left[ \frac{1}{(x+h)^2} - \frac{1}{x^2} \right] \\
 &= -\frac{2}{x^3} \\
 \left. \frac{dy}{dx} \right|_{x=-2} &= \frac{1}{4} \longrightarrow m_T = \frac{1}{4}
 \end{aligned}$$

$$\begin{aligned}
 4. \quad \frac{dy}{dx} = f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\frac{2}{(x+h)+3} - \frac{2}{x+3}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{1}{h} \left[ \frac{2}{(x+h)+3} - \frac{2}{x+3} \right] \\
 &= -\frac{2}{(x+3)^2} \\
 \left. \frac{dy}{dx} \right|_{x=-1} &= -\frac{1}{2} \longrightarrow m_T = -\frac{1}{2}
 \end{aligned}$$

5. Use the definition of derivative to get  $\frac{dy}{dt} = 40 - 32t$

Since  $\frac{dy}{dt} \Big|_{t=2} = -24 \longrightarrow$  the velocity at  $t = 2$  is  $-24$  feet per second.

6. Average Velocities

- On  $[3, 4] \longrightarrow \frac{s(4) - s(3)}{4 - 3} = -1$
- On  $[3.5, 4] \longrightarrow \frac{s(4) - s(3.5)}{4 - 3.5} = -.5$
- On  $[4, 5] \longrightarrow \frac{s(5) - s(4)}{5 - 4} = -1$
- On  $[4, 4.5] \longrightarrow \frac{s(4.5) - s(4)}{4.5 - 4} = .5$

Instantaneous velocity at  $t = 4$

$$\frac{ds}{dt} = 2t - 8$$

$\frac{ds}{dt} \Big|_{t=4} = 0 \longrightarrow$  the instantaneous velocity at  $t = 4$  is 0 meters per second.