1. False. Continuity does not imply differentiability.

2. True. Differentiability implies continuity.

3. False. Need product rule.

4. True.

5. True. The limit represents $f'(2)$ and since $f'(x) = 5x^4 \longrightarrow f'(2) = 80$.

6. False. The slope would be the value of the derivative at $x = 2$, not "$2x$".

7. $$\begin{aligned} f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \to 0} \frac{(x+h)^3 + 5(x+h) + 4 - (x^3 + 5x + 4)}{h} \\ &= \lim_{h \to 0} (3x^2 + 3xh + h^2 + 5) \\ &= 3x^2 + 5 \end{aligned}$$

8. $$\begin{aligned} f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \to 0} \frac{\sqrt{3 - 5(x+h)} - \sqrt{3 - 5x}}{h} \quad \text{(Rationalize the numerator.)} \\ &= -\frac{5}{2\sqrt{3 - 5x}} \end{aligned}$$

9. $$\begin{aligned} \frac{dy}{dx} &= (x+2)^8(6)(x+3)^5(1) + (x+3)^6(8)(x+2)^7(1) \\ &= 2(x+2)^7(x+3)^5(7x+18) \end{aligned}$$

10. $$\begin{aligned} \frac{dy}{dx} &= \frac{(9-4x)^{1/2}(1) - (x)\left(\frac{1}{2}\right)(9-4x)^{-1/2}(-4)}{9-4x} \\ &= \frac{9-2x}{\sqrt{(9-4x)^3}} \end{aligned}$$

11. $$\begin{aligned} f'(x) &= \frac{(8-3x)(1) - (x)(-3)}{(8-3x)^2} \\ &= \frac{8}{(8-3x)^2} \end{aligned}$$

12. $$\begin{aligned} \frac{dy}{dx} &= \frac{1}{5}(x \tan x)^{-4/5}[x \sec^2 x + \tan x] \\ &= \frac{x \sec^2 x + \tan x}{5\sqrt[5]{(x \tan x)^4}} \end{aligned}$$

13.
$$y = \frac{x^2-5x+4}{x^2-5x+6}$$
$$\frac{dy}{dx} = \frac{(x^2-5x+6)(2x-5)-(x^2-5x+4)(2x-5)}{(x^2-5x+6)^2}$$
$$= \frac{4x-10}{(x^2-5x+6)^2}$$

14.
$$g'(x) = \left[\sec^2\sqrt{1-x}\right]\left[\frac{1}{2}(1-x)^{-1/2}(-1)\right]$$
$$= -\frac{\sec^2\sqrt{1-x}}{2\sqrt{1-x}}$$

15.
$$\frac{dy}{dx} = \cos\left(\tan\sqrt{1-x^2}\right)\left(\sec^2\sqrt{1-x^2}\right)\left(\frac{1}{2}\right)(1-x^2)^{-1/2}(-2x)$$
$$= -\frac{x\cos\left(\tan\sqrt{1-x^2}\right)\left(\sec^2\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}$$

16. $h'(x) = -6x\csc^2(3x^2+5)$

17.
$$\frac{dy}{dx} = [2\cos(\tan x)][-\sin(\tan x)](\sec^2 x)$$
$$= -2\sec^2 x\cos(\tan x)\sin(\tan x)$$

18. $f'(x) = -\dfrac{10}{(2x-1)^6}$

$f''(x) = \dfrac{120}{(2x-1)^7} \longrightarrow f''(0) = -120$

19. Slope of tangent

$$\frac{dy}{dx} = \frac{-x^2-2}{(x^2-2)^2} \longrightarrow \left.\frac{dy}{dx}\right|_{x=2} = -\frac{3}{2}$$

Point

Given as $(2,1)$

Equation of tangent

$y-1 = -\dfrac{3}{2}(x-2)$

20. Slope of tangent

$f'(x) = \sec^2 x \longrightarrow f'\left(\dfrac{\pi}{3}\right) = 4$

Point

Given as $\left(\dfrac{\pi}{3}, \sqrt{3}\right)$

Equation of tangent

$y-\sqrt{3} = 4\left(x-\dfrac{\pi}{3}\right)$

21. $\dfrac{dy}{dx} = \cos x - \sin x$

$\dfrac{dy}{dx} = 0$ when $x = \dfrac{\pi}{4}$ or $x = \dfrac{5\pi}{4}$.

When $x = \dfrac{\pi}{4} \longrightarrow y = \sqrt{2}$

When $x = \dfrac{5\pi}{4} \longrightarrow y = -\sqrt{2}$

$\therefore$ the points where the tangent is horizontal are: $\left(\dfrac{\pi}{4}, \sqrt{2}\right)$ and $\left(\dfrac{5\pi}{4}, -\sqrt{2}\right)$

22. $\underline{h'(2)}$

$$\begin{aligned} h'(x) &= f(x)g'(x) + g(x)f'(x) \\ h'(2) &= f(2)g'(2) + g(2)f'2) \\ &= (3)(4) + (5)(-2) \\ &= 2 \end{aligned}$$

$\underline{F'(2)}$

$$\begin{aligned} F'(x) &= f'(g(x))g'(x) \\ F'(2) &= f'(g(2))g'(2) \\ &= f(5) \cdot 4 \\ &= 44 \end{aligned}$$

23. $f'(x) = x^2 g'(x) + g(x)2x = x^2 g'(x) + 2xg(x)$

24. $f'(x) = 2g(x)g'(x)$

25. $f'(x) = g'(g(x))g'(x)$

26. $$\begin{aligned} h'(x) &= \frac{[f(x)+g(x)][f(x)g'(x)+g(x)f'(x)] - [f(x)g(x)][f'(x)+g'(x)]}{[f(x)+g(x)]^2} \\ &= \frac{[f(x)]^2 g'(x) + [g(x)]^2 f'(x)}{[f(x)+g(x)]^2} \end{aligned}$$

27. $f(x) = x^6$ and the limit is $f'(2)$.

$f'(x) = 6x^5 \longrightarrow f'(2) = 192.$