

1. f is not differentiable at $x = -2$ because $f'_-(-2) \nexists$.

f is not differentiable at $x = 2$ because $f'_-(2) \neq f'_+(2) \therefore f'(2) \nexists$.

f is not differentiable at $x = 5$ because $f'_-(5) \neq f'_+(5) \therefore f'(5) \nexists$.

f is not differentiable at $x = 7$ because $f'_-(7) \neq f'_+(7) \therefore f'(7) \nexists$.

2. f is not differentiable at $x = 3$ because f is not continuous at $x = 3$.

f is not differentiable at $x = 4$ because $f'_-(4) \neq f'_+(4) \therefore f'(4) \nexists$.

f is not differentiable at $x = 6$ because $f'_-(6) \neq f'_+(6) \therefore f'(6) \nexists$.

$$3. f(x) = \begin{cases} x - 6 & x \geq 6 \\ 6 - x & x < 6 \end{cases}$$

$$f'(x) = \begin{cases} 1 & x > 6 \\ -1 & x < 6 \end{cases}$$

Since $f'_-(6) = -1$ but $f'_+(6) = 1 \rightarrow f'(6) \nexists \therefore f$ is not differentiable at $x = 6$.

4. f is not differentiable at any integer because f is not continuous at any integer.

5. Continuity test at $x = 0$

$$f(0) = 0$$

$$\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} (x - 1) = -1$$

Since $f(0) \neq \lim_{x \rightarrow 0} f(x)$, f is not continuous at $x = 0$ and therefore f is not differentiable at $x = 0$.

Continuity test at $x = 1$

$$f(1) = 0$$

$$\lim_{x \rightarrow 1^-} f(x) = 0 \text{ and } \lim_{x \rightarrow 1} f(x) = 0 \rightarrow \lim_{x \rightarrow 1} f(x) = 0$$

Since $f(1) = \lim_{x \rightarrow 1} f(x)$, f is continuous at $x = 1$

Differentiability at $x = 1$

$$f'(x) = \begin{cases} 1 & x < 1 \text{ and } x \neq 0 \\ 0 & x = 0 \\ -1 & x > 1 \end{cases}$$

Since $f'_-(1) = 1$ but $f'_+(1) = -1 \rightarrow f'(1) \nexists \therefore f$ is not differentiable at $x = 1$.

$$6. f'(x) = \begin{cases} 2x & x < 0 \\ 1 & x > 0 \end{cases}$$

Since $f'_-(0) = 0$ but $f'_+(0) = 1 \longrightarrow f'(0) \nexists \therefore f$ is not differentiable at $x = 0$.