1. $f'(x) = 2x - 10$
2. $V'(r) = 4\pi r^2$
3. $F(x) = 4096x^3 \longrightarrow F'(x) = 12288x^2$
4.

$$\begin{aligned} Y'(t) &= -54t^{-10} \\ &= -\frac{54}{t^{10}} \end{aligned}$$

5. $g(x) = x^2 + x^{-2}$

$$\begin{aligned} g'(x) &= 2x - 2x^{-3} \\ &= 2x - \frac{2}{x^3} \\ &= \frac{2x^4 - 2}{x^3} \end{aligned}$$

6.

$$\begin{aligned} h'(x) &= \frac{(x-1)(1) - (x+2)(1)}{(x-1)^2} \\ &= -\frac{3}{(x-1)^2} \end{aligned}$$

7. $G(s) = s^4 + s^3 + 3s^2 + 2s + 2 \longrightarrow G'(s) = 4s^3 + 3s^2 + 6s + 2$
8. $H(t) = t^{4/3} + 2t^{1/3}$

$$\begin{aligned} H'(t) &= \frac{4}{3}t^{1/3} + \frac{2}{3}t^{-2/3} \\ &= \frac{4\sqrt[3]{t}}{3} + \frac{2}{3\sqrt[3]{t^2}} \\ &= \frac{4t+2}{3\sqrt[3]{t^2}} \end{aligned}$$

9.

$$\begin{aligned} y &= x^{-1/2}\left(x^2 + 4x + 3\right) \\ &= x^{3/2} + 4x^{1/2} + 3x^{-1/2} \\ \frac{dy}{dx} &= \frac{3}{2}x^{1/2} + 2x^{-1/2} - \frac{3}{2}x^{-3/2} \\ &= \frac{3\sqrt{x}}{2} + \frac{2}{\sqrt{x}} - \frac{3}{2\sqrt{x^3}} \\ &= \frac{3x^2 + 4x - 3}{2\sqrt{x^3}} \end{aligned}$$

10. $y = \sqrt{5}x^{1/2}$

$$\frac{dy}{dx} = \frac{\sqrt{5}}{2}x^{-1/2}$$

$$= \frac{\sqrt{5}}{2\sqrt{x}}$$

11. $y = \dfrac{1}{x^4 + x^2 + 1}$

$$\frac{dy}{dx} = \frac{(x^4 + x^2 + 1)(0) - (1)(4x^3 + 2x)}{(x^4 + x^2 + 1)^2}$$

$$= -\frac{4x^3 + 2x}{(x^4 + x^2 + 1)^2}$$

12. $\dfrac{dy}{dx} = 2ax + b$

13. $$\frac{dy}{dt} = \frac{(t^2 + 5t - 4)(3) - (3t - 7)(2t + 5)}{(t^2 + 5t - 4)^2}$$

$$= \frac{23 + 14t - 3t^2}{(t^2 + 5t - 4)^2}$$

14. $y = x + x^{2/5}$

$$\frac{dy}{dx} = 1 + \frac{2}{5}x^{-3/5}$$

$$= 1 + \frac{2}{5\sqrt[5]{x^3}}$$

$$= \frac{5\sqrt[5]{x^3} + 2}{5\sqrt[5]{x^3}}$$

15. $f'(x) = \sqrt{2}x^{\sqrt{2}-1}$

16. $v = x^{3/2} + x^{-5/2}$

$$\frac{dv}{dx} = \frac{3}{2}x^{1/2} - \frac{5}{2}x^{-7/2}$$

$$= \frac{3x^4 - 5}{2\sqrt{x^7}}$$

17. $f(x) = \dfrac{x^2}{x^2 + c}$

$$f'(x) = \frac{(x^2 + c)(2x) - (x^2)(2x)}{(x^2 + c)^2}$$

$$= \frac{2cx}{(x^2 + c)^2}$$

18. $$f'(x) = \frac{(x^3 - 2)(5x^4) - (x^5)(3x^2)}{(x^3 - 2)^2}$$

$$= \frac{2x^7 - 10x^4}{(x^3 - 2)^2}$$

19. Point

    Given $(6, 2)$

    Slope

    $$\frac{dy}{dx} = -\frac{3}{(x-3)^2} \longrightarrow \left.\frac{dy}{dx}\right|_{x=6} = -\frac{1}{3} \longrightarrow m_T = -\frac{1}{3}$$

    Equation of Tangent

    $$y - 2 = -\frac{1}{3}(x - 6)$$

20. Point

    Given $(4, 32)$

    Slope

    $$\frac{dy}{dx} = \frac{5}{2}x^{3/2} \longrightarrow \left.\frac{dy}{dx}\right|_{x=4} = 20 \longrightarrow m_T = 20$$

    Equation of Tangent

    $$y - 32 = 20(x - 4)$$

21. Point

    When $x = 2 \longrightarrow y = 4 \longrightarrow (2, 4)$

    Slope

    $$\frac{dy}{dx} = \frac{x^2 - 4}{x^2} \longrightarrow \left.\frac{dy}{dx}\right|_{x=2} = 0 \longrightarrow m_T = 0$$

    Equation of Tangent

    $$y - 4 = 0(x - 2)$$

22. Slope from derivative

    $$\frac{dy}{dx} = \frac{2}{(x+1)^2}$$

    Slope from given line

    $$m = \frac{1}{2}$$

    Points

    $\frac{2}{(x+1)^2} = \frac{1}{2} \longrightarrow x = 1$ or $x = -3 \longrightarrow$ the points are $(1, 0)$ and $(-3, 2)$.

    Equations of tangents

    At $(1, 0) \longrightarrow y - 0 = \frac{1}{2}(x - 1)$.

    At $(-3, 2) \longrightarrow y - 2 = \frac{1}{2}(x + 3)$.

23. Slope from derivative

$$\frac{dy}{dx} = \frac{3\sqrt{x}}{2}$$

Slope of given line

$m = 3$

Points

$\dfrac{3\sqrt{x}}{2} = 3 \longrightarrow x = 4 \longrightarrow$ the point is $(4, 8)$

24. $f'(x) = 6x^2 - 6x - 6$

For the tangent to be horizontal, the slope of the tangent must be zero $\therefore$ we need $f'(x) = 0$

Now, $f'(x) = 0$ when $6x^2 - 6x - 6 = 0 \longrightarrow x = \dfrac{1+\sqrt{5}}{2}$ or $x = \dfrac{1-\sqrt{5}}{2}$.

25. $\dfrac{dy}{dx} = 3x^2 - 2x - 1$

For the tangent to be horizontal, the slope of the tangent must be zero $\therefore$ we need $\dfrac{dy}{dx} = 0$

Now, $\dfrac{dy}{dx} = 0$ when $3x^2 - 2x - 1 = 0 \longrightarrow x = 1$ or $x = -\dfrac{1}{3}$.

When $x = 1 \longrightarrow y = 0$ and when $x = -\dfrac{1}{3} \longrightarrow y = \dfrac{32}{27}$

$\therefore$ the points where the tangent is horizontal are $(1, 0)$ and $\left(-\dfrac{1}{3}, \dfrac{32}{27}\right)$

26. Note that the point $(2, -3)$ is *not* on the curve!

Slope from derivative

$\frac{dy}{dx} = 2x + 1$

Any point on the curve can be written $(x, x^2 + x)$. We will now find the slope of the tangent a second way by finding the slope of the line through $(2, -3)$ and $(x, x^2 + x)$.

$m = \frac{(x^2 + x) + 3}{x - 2}$

Slope from two points

$\frac{(x^2 + x) + 3}{x - 2} = 2x + 1 \longrightarrow x = 5 \text{ or } x = -1$

Points

When $x = 5 \longrightarrow y = 30 \longrightarrow (5, 30)$.
When $x = -1 \longrightarrow y = 0 \longrightarrow (-1, 0)$.

Slope of tangents

$\left.\frac{dy}{dx}\right|_{x=5} = 11 \longrightarrow m_T = 11 \text{ at } (5, 30)$

$\left.\frac{dy}{dx}\right|_{x=-1} = 11 \longrightarrow m_T = -1 \text{ at } (-1, 0)$

Equations of tangents

At $(5, 30) \longrightarrow y - 30 = 11(x - 5)$

At $(-1, 0) \longrightarrow y - 0 = -1(x + 1)$

27. Slope of normal

$\frac{dy}{dx} = -2x \longrightarrow \left.\frac{dy}{dx}\right|_{x=2} = -4 \longrightarrow m_\perp = \frac{1}{4}.$

Point

Given as $(2, -3)$

Equation of normal

$y + 3 = \frac{1}{4}(x - 2)$

28. <u>Slope of normal</u>

$$\frac{dy}{dx} = \frac{1}{3}x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}} \longrightarrow \left.\frac{dy}{dx}\right|_{x=-8} = \frac{1}{12} \longrightarrow m_{\perp} = -12.$$

<u>Point</u>

Given as $(-8, -2)$

<u>Equation of normal</u>

$y + 2 = -12(x + 8)$

29. <u>Slope of normal</u>

$$\frac{dy}{dx} = 4x^3$$

Since the slope of the tangent is given by $4x^3$, the slope of the normal is $-\dfrac{1}{4x^3}$.

Since we know the slope of the normal is 16,

$$-\frac{1}{4x^3} = 16 \longrightarrow x = -\frac{1}{4} \longrightarrow y = \frac{1}{256}.$$

Thus the point where the normal has slope 16 is $\left(-\dfrac{1}{4}, \dfrac{1}{256}\right)$.