1. $\lim_{x\to 0} (x^2 + \cos x) = 1$

2. $\lim_{x\to \pi/3} (\sin x - \cos x) = \dfrac{-1+\sqrt{3}}{2}$

3. $\lim_{t\to \pi/4} \dfrac{\sin 5t}{t} = -\dfrac{2\sqrt{2}}{\pi}$

4. $\lim_{x\to 0} \dfrac{\sin(\cos x)}{\sec x} = \sin 1$

5. $\lim_{x\to \pi/4} \dfrac{\sin x}{3x} = \dfrac{2\sqrt{2}}{3\pi}$

6. $\lim_{x\to \pi/4} \dfrac{\tan x}{4x} = \dfrac{1}{\pi}$

7. $\lim_{x\to 0} \dfrac{\sin^2 x}{x} = \lim_{x\to 0} \dfrac{\sin x}{x} \sin x = 0$

8. $$\begin{aligned}\lim_{x\to 0} \frac{\tan 3x}{3\tan 2x} &= \lim_{x\to 0} \frac{\frac{\sin 3x}{\cos 3x}}{\frac{3\sin 2x}{\cos 2x}} \\ &= \lim_{x\to 0} \frac{\sin 3x}{\cos 3x}\, \frac{\cos 2x}{3\sin 2x} \\ &= \lim_{x\to 0} \frac{\frac{\sin 3x}{3x}3x}{\cos 3x}\, \frac{\cos 2x}{\frac{3\sin 2x}{2x}2x} \\ &= \frac{1}{2}\end{aligned}$$

9. $\dfrac{dy}{dx} = -\sin x - 2\sec^2 x$

10. $\dfrac{dy}{dx} = \cos x - \sin x$

11. $$\begin{aligned}\frac{dy}{dx} &= (x)(-\csc x \cot x) + (\csc x)(1) \\ &= \csc x - x \csc x \cot x\end{aligned}$$

12. $$\begin{aligned}\frac{dy}{dx} &= (\csc x)(-\csc^2 x) + (\cot x)(-\csc x \cot x) \\ &= -\csc^3 x - \cot^2 x \csc x \\ &= -\csc x(\csc^2 x + \cot^2 x)\end{aligned}$$

13. $$\begin{aligned}\frac{dy}{dx} &= \frac{(1+\cos x)(\cos x) - (\sin x)(-\sin x)}{(1+\cos x)^2} \\ &= \frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2} \\ &= \frac{1+\cos x}{(1+\cos x)^2} \\ &= \frac{1}{1+\cos x}\end{aligned}$$

14. $\dfrac{dy}{dx} = \dfrac{x\sec^2 x - \tan x}{x^2}$

15. $$\begin{aligned}\frac{dy}{dx} &= \frac{(\sin x + \cos x)(1) - x(\cos x - \sin x)}{(\sin x + \cos x)^2} \\ &= \frac{\sin x + \cos x - x\cos x + x\sin x}{(\sin x + \cos x)^2}\end{aligned}$$

16. $$\begin{aligned}\frac{dy}{dx} &= (x^{-3})(\sin x \sec^2 x + \tan x \cos x) + (\sin x \tan x)(-3x^{-4}) \\ &= \frac{\sin x \sec^2 x + \tan x \cos x}{x^3} + \frac{-3\sin x \tan x}{x^4} \\ &= \frac{x\sin x \sec^2 x + x\tan x \cos x - 3\sin x \tan x}{x^4}\end{aligned}$$

17. $$\begin{aligned}\frac{dy}{dx} &= \frac{(\sec x)\left[(x^2)(\sec^2 x) + (\tan x)(2x)\right] - (x^2 \tan x)(\sec x \tan x)}{\sec^2 x} \\ &= \frac{x^2\sec^3 x + 2x\sec x \tan x - x^2 \sec x \tan^2 x}{\sec^2 x}\end{aligned}$$

18. Slope of tangent

$\dfrac{dy}{dx} = x(-\sin x) + \cos x \longrightarrow \left.\dfrac{dy}{dx}\right|_{x=\pi} = -1 \longrightarrow m_T = -1$

Point

Given as $(\pi, -\pi)$

Equation of tangent

$y + \pi = -(x - \pi)$

19. Slope of tangent

$\dfrac{dy}{dx} = \sec^2 x \longrightarrow \left.\dfrac{dy}{dx}\right|_{x=\pi/4} = 2 \longrightarrow m_T = 2$

Point

Given as $\left(\dfrac{\pi}{4}, 1\right)$

Equation of tangent

$y - 1 = 2\left(x - \dfrac{\pi}{4}\right)$

20. Slope of tangent

$$g'(x) = 2\cos x \longrightarrow g'\left(\frac{\pi}{6}\right) = \sqrt{3} \longrightarrow m_T = \sqrt{3}$$

Point

$$g\left(\frac{\pi}{6}\right) = 1 \longrightarrow \left(\frac{\pi}{6}, 1\right)$$

Equation of tangent

$$y - 1 = \sqrt{3}\left(x - \frac{\pi}{6}\right)$$

21. For $f$ to have a horizontal tangent, $f'(x) = 0$.

$$f'(x) = 1 + 2\cos x$$

$$f'(x) = 0 \text{ when } 1 + 2\cos x = 0 \longrightarrow x = \frac{2\pi}{3} + 2k\pi \text{ or } x = \frac{4\pi}{3} + 2k\pi$$