1. $\dfrac{dy}{du} = 2u$ and $\dfrac{du}{dx} = 2x + 2$

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du} \cdot \frac{du}{dx} \\ &= (2u)(2x+2) \\ &= 2(x^2+2x+3)(2x+2)\end{aligned}$$

2. $\dfrac{dy}{du} = 2u - 2$ and $\dfrac{du}{dx} = -6$

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du} \cdot \frac{du}{dx} \\ &= (2u-2)(-6) \\ &= -12(4-6x)\end{aligned}$$

3. $\dfrac{dy}{du} = 3u^2$ and $\dfrac{du}{dx} = 1 - x^{-2}$

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du} \cdot \frac{du}{dx} \\ &= (3u^2)(1-x^{-2}) \\ &= 3\left(x + \frac{1}{x}\right)^2 \left(1 - \frac{1}{x^2}\right)\end{aligned}$$

4. $\dfrac{dy}{du} = 1 - 2u$ and $\dfrac{du}{dx} = \dfrac{1}{2}x^{-1/2} + \dfrac{1}{3}x^{-2/3}$

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du} \cdot \frac{du}{dx} \\ &= (3u^2)(1-x^{-2}) \\ &= \left(1 - 2\left(x^{1/2} + x^{1/3}\right)\right)\left(\frac{1}{2}x^{-1/2} + \frac{1}{3}x^{-2/3}\right)\end{aligned}$$

5. $F'(x) = 2(x^2+4x+6)(2x+4)$

6. $G'(x) = 4(x^3-5x)^3(3x^2-5)$

7. $g'(x) = (3x-2)^{10}\left[12(5x^2-x+1)^{11}\right](10x-1) + (5x^2-x+1)^{12}\left[10(3x-2)^9\right]3$

8. $$\begin{aligned}f'(t) &= -8(2t^2-6t+1)^{-9}(4t-6) \\ &= -\frac{8(4t-6)}{(2t^2-6t+1)^9}\end{aligned}$$

9. $g'(x) = \dfrac{2x-7}{2\sqrt{x^2-7x}}$

10. $$\begin{aligned} h'(t) &= \frac{3}{2}\left(t-\frac{1}{t}\right)^{1/2}(1+t^{-2}) \\ &= \frac{3\sqrt{t-\frac{1}{t}}\left(1+\frac{1}{t^2}\right)}{2} \\ &= \frac{3\sqrt{t-\frac{1}{t}}+\frac{3}{t^2}\sqrt{1-\frac{1}{t}}}{2} \\ &= \frac{3t^2\sqrt{t-\frac{1}{t}}+3\sqrt{1-\frac{1}{t}}}{2t^2} \end{aligned}$$

11. $$\begin{aligned} f'(y) &= 3\left(\frac{y-6}{y+7}\right)^2\left[\frac{(y+7)(1)-(y-6)(1)}{(y+7)^2}\right] \\ &= 3\left(\frac{y-6}{y+7}\right)^2\left[\frac{13}{(y+7)^2}\right] \end{aligned}$$

12. $s'(t) = \frac{1}{4}\left(\frac{t^3+1}{t^3-1}\right)^{-3/4}\left[\frac{(t^3-1)(3t^2)-(t^3+1)(3t^2)}{(t^3-1)^2}\right]$ No need to simplify further!

13. $f(z) = (2z-1)^{-1/5}$

$$\begin{aligned} f'(z) &= -\frac{1}{5}(2z-1)^{-6/5}(2) \\ &= -\frac{2}{5\sqrt[5]{(2z-1)^6}} \end{aligned}$$

14. $\frac{dy}{dx} = 3\sec^2 3x$

15. $\frac{dy}{dx} = -3x^2\sin(x^3)$

16. $\frac{dy}{dx} = -3\cos^2 x\sin x$

17. $\frac{dy}{dx} = 6(1+\cos^2 x)^5(-2\cos x\sin x) = [-12\sin x\cos x](1+\cos^2 x)^5$

18. $\frac{dy}{dx} = -\sec^2 x\sin(\tan x)$

19. $$\begin{aligned} f'(x) &= (2\sec 2x)(\sec 2x\tan 2x)(2)-(2\tan 2x)(\sec^2 2x)(2) \\ &= 4\sec^2 2x\tan 2x - 4\sec^2 2x\tan 2x \\ &= 0 \end{aligned}$$

Note: $1+\tan^2 x = \sec^2 x \longrightarrow$ so $f(x) = 0$ to begin with and the derivative of a constant is zero!

20. $f'(x) = -\frac{1}{3}\left(\csc\frac{1}{3}x\right)\left(\cot\frac{1}{3}x\right)$

21. $\frac{dy}{dx} = 3\sin^2 x\cos x - 3\cos^2 x\sin x$

22. $p'(x) = \left(\cos\frac{1}{x}\right)\left(-\frac{1}{x^2}\right) = -\frac{1}{x^2}\cos\frac{1}{x}$

23. $$\frac{dy}{dx} = \frac{(1-\sin x)(\cos x)-(1+\sin x)(-\cos x)}{(1-\sin x)^2}$$
$$= \frac{2\cos x}{(1-\sin x)^2}$$

24. $$\frac{dy}{dx} = \left(2\tan x^2\right)\left(\sec^2 x^2\right)(2x)$$
$$= 4x\tan x^2 \sec^2 x^2$$

25. $$\frac{dy}{dx} = \frac{1}{2}\left(x+\sqrt{x}\right)^{-1/2}\left(1+\frac{1}{2\sqrt{x}}\right)$$
$$= \frac{1}{2\sqrt{x+\sqrt{x}}}\left(1+\frac{1}{2\sqrt{x}}\right)$$

26. Slope of tangent

$$\frac{dy}{dx} = 10(x^3-x^2+x-1)^9(3x^2-2x+1)$$
$$\left.\frac{dy}{dx}\right|_{x=1} = 0 \longrightarrow m_T = 0$$

Point

Given as $(1,0)$

Equation of tangent

$y-0=0(x-1)$

27. Slope of tangent

$$f'(x) = -\frac{12}{\sqrt{(3x+4)^3}}$$
$$f'(4) = -\frac{3}{16} \longrightarrow m_T = -\frac{3}{16}$$

Point

Given as$(4,2)$

Equation of tangent

$$y-2=-\frac{3}{16}(x-4)$$

28. $f'(x) = 2\cos x + 2\sin x \cos x = 2\cos x(1 + \sin x)$

In order to have a horizontal tangent, $f'(x) = 0$.

Now, $f'(x) = 0$ when $2\cos x(1 + \sin x) = 0$

This will be true when $\cos x = 0$ or when $\sin x = -1$.

$\cos x = 0$ when $x = \frac{\pi}{2}$ or $x = \frac{3\pi}{2}$

$\sin x = -1$ when $x = \frac{3\pi}{2}$

$f\left(\frac{\pi}{2}\right) = 3$ and $f\left(\frac{3\pi}{2}\right) = -1$

These function values will be the same for any multiple of $\frac{\pi}{2}$ or $\frac{3\pi}{2}$

$\therefore$ the points where the tangent is horizontal are: $\left(\frac{\pi}{2} + 2k\pi, 3\right)$ and $\left(\frac{3\pi}{2} + 2k\pi, -1\right)$

29. We know that $F'(x) = f'(g(x))g'(x) \longrightarrow F'(3) = f'(g(3))g'(3)$

So $F'(3) = f'(6) \cdot 4$

$\therefore F'(3) = (7)(4) = 28.$