

AP CALCULUS
TRIGONOMETRIC LIMITS

$$1. \lim_{x \rightarrow 0} \frac{\sin 4x}{x} = \lim_{x \rightarrow 0} \frac{\frac{\sin 4x}{4x} 4x}{x} = 4$$

$$2. \lim_{x \rightarrow 0} \frac{\sin 4x}{\sin 7x} = \lim_{x \rightarrow 0} \frac{\frac{\sin 9x}{9x} 9x}{\frac{\sin 7x}{7x} 7x} = \frac{9}{7}$$

$$3. \lim_{x \rightarrow 0} \frac{3x}{\sin 5x} \lim_{x \rightarrow 0} \frac{3x}{\frac{\sin 5x}{5x} 5x} = \frac{3}{5}$$

$$4. \lim_{x \rightarrow 0} \frac{x^2}{\sin^2 3x} = \lim_{x \rightarrow 0} \frac{\frac{x^2}{3x} \frac{x^2}{\sin 3x} \sin 3x}{\frac{x^2}{3x}} = \frac{1}{9}$$

$$5. \lim_{x \rightarrow 0} \frac{x}{\cos x} = 0$$

Just plug in the 0 and you get a number out! Remember, the first step in finding any limit is to plug in the number to see if you get a number out—if you do, you’re done!

$$6. \lim_{x \rightarrow 0} \frac{1 - \cos 2x}{4x} = \lim_{x \rightarrow 0} \frac{\frac{1 - \cos 2x}{2x} 2x}{4x} = (0) \left(\frac{1}{2} \right) = 0$$

$$7. \lim_{x \rightarrow 0} \frac{\tan x}{2x} = \lim_{x \rightarrow 0} \frac{\frac{\sin x}{\cos x} \frac{1}{2x}}{2x} = \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} x \right) \left(\frac{1}{\cos x} \right) \left(\frac{1}{2x} \right) = \frac{1}{2}$$

$$8. \lim_{x \rightarrow 0} \frac{1 - \cos 8x}{\sin 3x} = \lim_{x \rightarrow 0} \frac{\frac{1 - \cos 8x}{8x} 8x}{\frac{\sin 3x}{3x} 3x} = \frac{(0)(8)}{(1)(3)} = 0$$

$$9. \lim_{x \rightarrow 0} \frac{x^2 + 3x}{\sin x} = \lim_{x \rightarrow 0} \frac{\frac{x(x+3)}{\sin x} x}{\frac{\sin x}{x} x} = 3$$