1. True by the following theorem: $\lim_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$

2. False.

   Consider $f(x) = \begin{cases} x^2+1 & x \neq 0 \\ 2 & x = 0 \end{cases}$

   $f(x) > 1 \ \forall\ x$ but $\lim_{x\to 0} f(x) = 1$ which is *not* greater than 1.

3. True by the definition of limit.

4. True. . . it's a theorem!

5. $\lim_{x\to 4} \sqrt{x+\sqrt{x}} = \sqrt{6}$

6. $\lim_{t\to -1} \dfrac{t+1}{t^3-t} = \lim_{t\to -1} \dfrac{t+1}{t(t+1)(t-1)} = \lim_{t\to -1} \dfrac{1}{t(t-1)} = \dfrac{1}{2}$

7. $\lim_{h\to 0} \dfrac{(1+h)^2-1}{h} = \lim_{h\to 0} \dfrac{h(2+h)}{h} = \lim_{h\to 0} (2+h) = 2$

8. $\lim_{x\to -1} \dfrac{x^2-x-2}{x^2+3x-2} = \lim_{x\to -1} \dfrac{(x-2)(x+1)}{x^2+3x-2} = \dfrac{0}{-4} = 0$

9. $\lim_{t\to 0} \dfrac{17}{(t-6)^2} = \dfrac{17}{36}$

10. $\lim_{s\to 16} \dfrac{4-\sqrt{s}}{s-16} = \lim_{s\to 16} \dfrac{4-\sqrt{s}}{\left(\sqrt{s}+4\right)\left(\sqrt{s}-4\right)} = -\dfrac{1}{8}$

11. Since $\dfrac{|x-8|}{x-8} = \begin{cases} \dfrac{x-8}{x-8} & x > 8 \\[2ex] \dfrac{8-x}{x-8} & x < 8 \end{cases}$, $\lim_{t\to 8^-} \dfrac{|x-8|}{x-8} = \lim_{t\to 8^-} \dfrac{8-x}{x-8} = -1$

    We only need the one limit because the problem only asked for a left-hand limit.

12. $\lim_{x\to 0} \dfrac{1-\sqrt{1-x^2}}{x} = \lim_{x\to 0} \dfrac{1-\sqrt{1-x^2}}{x} \dfrac{1+\sqrt{1-x^2}}{1+\sqrt{1-x^2}} = \lim_{x\to 0} \dfrac{1-(1-x^2)}{x\left(1+\sqrt{1-x^2}\right)} = \lim_{x\to 0} \dfrac{x}{1+\sqrt{1-x^2}} = 0$

13. $\lim_{x\to \infty} \dfrac{1+2x-x^2}{1-x+2x^2} = -\dfrac{1}{2}$

14. $\lim_{x\to \infty} \dfrac{\sqrt{x^2-9}}{2x-6} = \dfrac{1}{2}$

15. $\lim_{x\to -\infty} \dfrac{\sqrt{x^2-9}}{2x-6} = -\dfrac{1}{2}$

16. $\lim_{x\to \infty} \left(\sqrt[3]{x} - \dfrac{x}{3}\right) = \lim_{x\to \infty} \dfrac{3\sqrt[3]{x}-x}{3} = -\infty$

17. For $\forall\ \epsilon > 0$ we need to find a $\delta > 0$ such that whenever

$$\begin{aligned} |x-5| < \delta \ \longrightarrow \ |(7x-27)-8| &< \epsilon \\ |7x-35| &< \epsilon \\ |x-5| &< \frac{\epsilon}{7} \end{aligned}$$

$$\therefore \text{choose } \delta = \min\left\{1, \frac{\epsilon}{7}\right\}.$$

18. For $\forall\ \epsilon > 0$ we need to find a $\delta > 0$ such that whenever

$$\begin{aligned} |x-2| < \delta \ \longrightarrow \ \left|(x^2-3x)+2\right| &< \epsilon \\ |x^2-3x+2| &< \epsilon \\ |x-2||x-1| &< \epsilon \\ |x-2| &< \frac{\epsilon}{|x-1|} \end{aligned}$$

Consider the interval $[1,3]$.

$$\text{When } x = 1 \ \longrightarrow \ \frac{\epsilon}{|x-1|} \nexists$$

$$\text{When } x = 3 \ \longrightarrow \ \frac{\epsilon}{|x-1|} = \frac{\epsilon}{2}$$

$$\therefore \text{choose } \delta = \min\left\{1, \frac{\epsilon}{2}\right\}.$$

19. (a) 3
    (b) 0
    (c) $\nexists$
    (d) 0
    (e) 0
    (f) 0

20. Let $f(x) = 2x^3 + x^2 + 2$.

    Since $f(-2) = -10 < 0$ and $f(-1) = 1 > 0 \ \longrightarrow \ f(x) = 0$ for some $x \in (-2,-1)$
    $\therefore\ 2x^3 + x^2 + 2 = 0$ has a root on $(-2,-1)$.

21. Since $f(3) < 0$ and $f(5) > 0$ then there exists *at least one* $x \in (3,5)$ such that $f(x) = 0$. In other words, since $f(3) < 0$ and $f(5) > 0$, we are guaranteed at least one zero on $(3,5)$.