1. Since the degree of the numerator is smaller than the degree of the denominator, $\lim_{x\to\infty} \frac{1}{x\sqrt{x}} = 0$.

2. Since the degree of the numerator is smaller than the degree of the denominator, $\lim_{x\to\infty} \frac{x+4}{x^2-2x+5} = 0$.

3. First, multiply the binomials out.

   Since the degree of the numerator and denominator are equal we divide the coefficients of the highest degreed terms and get $\lim_{x\to\infty} \frac{-x^2-x+2}{-6x^2+x+2} = \frac{1}{6}$.

4. Since the degree of the numerator is smaller than the degree of the denominator, $\lim_{x\to\infty} \frac{1}{3+\sqrt{x}} = 0$.

5. Since the degree of the numerator is smaller than the degree of the denominator, $\lim_{r\to\infty} \frac{r^4-r^2+1}{r^5+r^3-r} = 0$.

6. Since the degree of the numerator and denominator are equal we carefully divide the coefficients of the highest degreed terms and get $\lim_{x\to\infty} \frac{\sqrt{1+4x^2}}{4+x} = 2$.

7. Since the degree of the numerator and denominator are equal we carefully divide the coefficients of the highest degreed terms and get $\lim_{x\to\infty} \frac{1-\sqrt{x}}{1+\sqrt{x}} = -1$.

8. $\lim_{x\to\infty} \left(\sqrt{x^2+1}-\sqrt{x^2-1}\right) = 0$

9. $\lim_{x\to\infty} \left(\sqrt{1+x}-\sqrt{x}\right) = 0$

10. $\lim_{x\to\infty} \sqrt{x} = \infty \quad (\nexists)$

11. $\lim_{x\to\infty} \left(x-\sqrt{x}\right) = \infty \quad (\nexists)$

12. $\lim_{x\to-\infty} \left(x^3-5x^2\right) = -\infty \quad (\nexists)$

13. Since the degree of the numerator is greater than the degree of the denominator, the limit will not exist. (The value of the expression will go to $\pm\infty$).

    $\lim_{x\to\infty} \frac{x^7-1}{x^6+1} = \infty \quad (\nexists)$

14. Since the degree of the numerator is smaller than the degree of the denominator, $\lim_{x\to\infty} \frac{\sqrt{x}+3}{x+3} = 0$.

15. $\lim_{x\to-\infty} \frac{\sqrt{x}+3}{x+3}$ $\nexists$ because we cannot take the square root of a negative number.

16. Vertical Asymptotes

    Since $\frac{x}{x+4}$ $\nexists$ at $x=-4$ and $\lim_{x\to-4} \frac{x}{x+4} = \pm\infty$ there is a vertical asymptote at $x=-4$.

    Horizontal Asymptotes

    Since $\lim_{x\to\infty} \frac{x}{x+4} = 1$ there is a horizontal asymptote at $y=1$.

    Since $\lim_{x\to-\infty} \frac{x}{x+4} = 1$ there is a horizontal asymptote at $y=1$.

17. Vertical Asymptotes

Since $\dfrac{x^3}{x^2+3x-10}$ $\nexists$ at $x=-5$ and $\lim\limits_{x\to -5}\dfrac{x^3}{x^2+3x-10}=\pm\infty$ there is a vertical asymptote at $x=-5$.

Since $\dfrac{x^3}{x^2+3x-10}$ $\nexists$ at $x=2$ and $\lim\limits_{x\to 2}\dfrac{x^3}{x^2+3x-10}=\pm\infty$ there is a vertical asymptote at $x=2$.

Horizontal Asymptotes

Since $\lim\limits_{x\to\infty}\dfrac{x^3}{x^2+3x-10}=\infty$ there is *no* horizontal asymptote .

Since $\lim\limits_{x\to-\infty}\dfrac{x^3}{x^2+3x-10}=-\infty$ there is *no* horizontal asymptote .

18. Vertical Asymptotes

Since $\dfrac{x}{\sqrt[4]{x^4+1}}$ $\exists\ \forall\ x$ $\therefore$ there are no vertical asymptotes.

Horizontal Asymptotes

Since $\lim\limits_{x\to\infty}\dfrac{x}{\sqrt[4]{x^4+1}}=1$ there is a horizontal asymptote at $y=1$.

Since $\lim\limits_{x\to-\infty}\dfrac{x}{\sqrt[4]{x^4+1}}=-1$ there is a horizontal asymptote at $y=-1$.

19. Vertical Asymptotes

Since $\dfrac{x^2-25}{x-5}$ $\nexists$ at $x=5$ but $\lim\limits_{x\to 5}\dfrac{x^2-25}{x-5}=10$ there is *no* vertical asymptote at $x=5$.

Horizontal Asymptotes

Since $\lim\limits_{x\to\infty}\dfrac{x^2-25}{x-5}=\infty$ there is *no* horizontal asymptote .

Since $\lim\limits_{x\to-\infty}\dfrac{x^2-25}{x-5}=-\infty$ there is *no* horizontal asymptote .

20. Vertical Asymptotes

Since $\dfrac{x-3}{x+2}$ $\nexists$ at $x=-2$ and $\lim\limits_{x\to 2}\dfrac{x-3}{x+2}=\pm\infty$ there is a vertical asymptote at $x=-2$.

Horizontal Asymptotes

Since $\lim\limits_{x\to\infty}\dfrac{x-3}{x+2}=1$ there is a horizontal asymptote at $y=1$.

Since $\lim\limits_{x\to-\infty}\dfrac{x-3}{x+2}=1$ there is a horizontal asymptote at $y=1$.