1. For $\epsilon = .1$ we need to find a δ such that whenever

$$\begin{aligned} |x-3| < \delta \longrightarrow |(6x+1) - 19| &< .1 \\ |6x - 18| &< .1 \\ 6|x-3| &< .1 \\ |x-3| &< \frac{.1}{6} \end{aligned}$$

$$\therefore \text{choose } \delta \leq \frac{.1}{6}.$$

2. For $\epsilon = .5$ we need to find a δ such that whenever

$$\begin{aligned} |x-2| < \delta \longrightarrow \left|\frac{1}{x} - \frac{1}{2}\right| &< \frac{1}{2} \\ \left|\frac{2-x}{2x}\right| &< \frac{1}{2} \\ |x-2| &< |x| \end{aligned}$$

Consider the interval $[1, 3]$.

When $x = 1 \longrightarrow |x| = 1$
When $x = 3 \longrightarrow |x| = 3$
$\therefore$ choose $\delta \leq 1$.

3. For $\epsilon = .5$ we need to find a δ such that whenever

$$\begin{aligned} |x-1| < \delta \longrightarrow \left|x^2 - 1\right| &< .5 \\ |x-1||x+1| &< .5 \\ |x-1| &< \frac{.5}{|x+1|} \end{aligned}$$

Consider the interval $[0, 2]$.

When $x = 0 \longrightarrow \frac{.5}{|x+1|} = .5$
When $x = 2 \frac{.5}{|x+1|} = \frac{.5}{3}$
$\therefore$ choose $\delta \leq \frac{.5}{3}$.

4. For $\forall\ \epsilon > 0$ we need to find a $\delta > 0$ such that whenever

$$\begin{aligned} |x-2| < \delta \longrightarrow |(3x-2)-4| &< \epsilon \\ |3x-6| &< \epsilon \\ |x-2| &< \frac{\epsilon}{3} \\ \therefore \text{choose } \delta = \min \left\{1, \frac{\epsilon}{3}\right\}. \end{aligned}$$

5. For $\forall\ \epsilon > 0$ we need to find a $\delta > 0$ such that whenever

$$\begin{aligned} |x+1| < \delta \longrightarrow |(5x+8)-3| &< \epsilon \\ |5x+5| &< \epsilon \\ |x+1| &< \frac{\epsilon}{5} \\ \therefore \text{choose } \delta = \min \left\{1, \frac{\epsilon}{5}\right\}. \end{aligned}$$

6. For $\forall\ \epsilon > 0$ we need to find a $\delta > 0$ such that whenever

$$\begin{aligned} |x-2| < \delta \longrightarrow \left|\frac{x}{7} - \frac{2}{7}\right| &< \epsilon \\ \frac{1}{7}|x-2| &< \epsilon \\ |x-2| &< 7\epsilon \\ \therefore \text{choose } \delta = \min \{1, 7\epsilon\}. \end{aligned}$$

7. For $\forall\ \epsilon > 0$ we need to find a $\delta > 0$ such that whenever

$$\begin{aligned} |x+5| < \delta \longrightarrow \left|\left(4 - \frac{3x}{5}\right) - 7\right| &< \epsilon \\ \left|\frac{-15-3x}{5}\right| &< \epsilon \\ \left|\frac{-3(x+5)}{5}\right| &< \epsilon \\ \frac{3}{5}|x+5| &< \epsilon \\ |x+5| &< \frac{5\epsilon}{3} \\ \therefore \text{choose } \delta = \min \left\{1, \frac{5\epsilon}{3}\right\}. \end{aligned}$$

8. For $\forall\ \epsilon > 0$ we need to find a $\delta > 0$ such that whenever

$$|x-a| < \delta \longrightarrow |x-a| < \epsilon$$
$$\therefore \text{choose } \delta = \min\ \{1, \epsilon\}.$$

9. For $\forall\ \epsilon > 0$ we need to find a $\delta > 0$ such that whenever

$$|x-0| < \delta \longrightarrow \left|x^2-0\right| < \epsilon$$
$$|x-0||x+0| < \epsilon$$
$$|x-0| < \frac{\epsilon}{|x|}$$

Consider the interval $[-1, 1]$.

When $x = -1 \longrightarrow \frac{\epsilon}{|x|} = \epsilon$

When $x = 1 \longrightarrow \frac{\epsilon}{|x|} = \epsilon$

$\therefore$ choose $\delta = \min\ \{1, \epsilon\}$.

10. For $\forall\ \epsilon > 0$ we need to find a $\delta > 0$ such that whenever

$$|x-2| < \delta \longrightarrow \left|(x^2-4x+5)-1\right| < \epsilon$$
$$|x^2-4x+4| < \epsilon$$
$$|x-2||x-2| < \epsilon$$
$$|x-2| < \frac{\epsilon}{|x-2|}$$

Consider the interval $[1, 3]$.

When $x = 1 \longrightarrow \frac{\epsilon}{|x-2|} = \epsilon$

When $x = 3 \longrightarrow \frac{\epsilon}{|x-2|} = \epsilon$

$\therefore$ choose $\delta = \min\ \{1, \epsilon\}$.

11. For $\forall\ \epsilon > 0$ we need to find a $\delta > 0$ such that whenever

$$\begin{aligned} |x+2| < \delta \ \longrightarrow \ \left|(x^2-1)-3\right| &< \epsilon \\ |x^2-4| &< \epsilon \\ |x-2||x+2| &< \epsilon \\ \\ |x+2| &< \frac{\epsilon}{|x-2|} \end{aligned}$$

Consider the interval $[-3,-1]$.

$$\text{When } x=-3 \ \longrightarrow \ \frac{\epsilon}{|x-2|} = \frac{\epsilon}{5}$$

$$\text{When } x=-1 \ \longrightarrow \ \frac{\epsilon}{|x-2|} = \frac{\epsilon}{3}$$

$$\therefore \text{choose } \delta = \min\left\{1, \frac{\epsilon}{5}\right\}.$$

12. For $\forall\ \epsilon > 0$ we need to find a $\delta > 0$ such that whenever

$$\begin{aligned} |x-3| < \delta \ \longrightarrow \ \left|(x^2+5x+6)-30\right| &< \epsilon \\ |x^2+5x-24| &< \epsilon \\ |x-3||x+8| &< \epsilon \\ \\ |x-3| &< \frac{\epsilon}{|x+8|} \end{aligned}$$

Consider the interval $[2,4]$.

$$\text{When } x=2 \ \longrightarrow \ \frac{\epsilon}{|x+8|} = \frac{\epsilon}{10}$$

$$\text{When } x=4 \ \longrightarrow \ \frac{\epsilon}{|x+8|} = \frac{\epsilon}{12}$$

$$\therefore \text{choose } \delta = \min\left\{1, \frac{\epsilon}{12}\right\}.$$

13. For $\forall\ \epsilon > 0$ we need to find a $\delta > 0$ such that whenever

$$\begin{aligned} |x-3| < \delta \ \longrightarrow \ \left|(x^2+x-4)-8\right| &< \epsilon \\ |x^2+x-12| &< \epsilon \\ |x+4||x-3| &< \epsilon \\ \\ |x-3| &< \frac{\epsilon}{|x+4|} \end{aligned}$$

Consider the interval $[2,4]$.

$$\text{When } x=2 \ \longrightarrow \ \frac{\epsilon}{|x+4|} = \frac{\epsilon}{6}$$

$$\text{When } x=4 \ \longrightarrow \ \frac{\epsilon}{|x+4|} = \frac{\epsilon}{8}$$

$$\therefore \text{choose } \delta = \min\left\{1, \frac{\epsilon}{8}\right\}.$$