1. First of all, ignore the endpoints.
   - Discontinuous at $x = -3$, because $f(-3)$ does not exist.
   - Discontinuous at $x = 1$ because $f(1) \neq \lim_{x\to 1} f(x)$ does not exist.
   - Discontinuous at $x = 3$ because $f(3)$ does not exist.
2. <u>Test for continuity at $x = 3$</u>
   i. $f(3) = -48$
   ii. $\lim_{x\to 3} f(x) = -48$

   Since $f(3) = \lim_{x\to 3} f(x)$, f is continuous at $x = 3$.
3. <u>Test for continuity at $x = 5$</u>
   i. $f(5) = 5$
   ii. $\lim_{x\to 5} f(x) = 5$

   Since $f(5) = \lim_{x\to 5} f(x)$, f is continuous at $x = 5$.
4. <u>Test for continuity at $t = -8$</u>
   i. $f(-8) = -\dfrac{2}{2401}$
   ii. $\lim_{t\to -8} f(t) = -\dfrac{2}{2401}$

   Since $f(-8) = \lim_{t\to -8} f(t)$, f is continuous at $t = -8$.
5. Since $f(1) \nexists$, f is discontinuous at $x = 1$.
6. Since $\lim_{x\to 1} f(x) \nexists$, f is discontinuous at $x = 1$.
7. $f(3) = 5$ but $\lim_{x\to 3} f(x) = 7 \;\therefore\; f(3) \neq \lim_{x\to 3} f(x) \;\therefore\; f$ is discontinuous at $x = 3$.
8. The only place where f might be discontinuous is at $x = 3$.

   <u>Test for continuity at $x = 3$</u>
   i. $f(3) = 3c + 1$
   ii. $\lim_{x\to 3^+} f(x) = 9c - 1$ and $\lim_{x\to 3^-} f(x) = 3c + 1$

   For f to be continuous at $x = 3 \;\longrightarrow\; 3c + 1 = 9c - 1 \;\longrightarrow\; c = \dfrac{1}{3}$.

9. The only place where h might be discontinuous is at $x = 1$ and $x = 2$.

   <u>Test for continuity at $x = 1$</u>

   i. $h(1) = c + d$

   ii. $\lim_{x \to 1^+} h(x) = c + d$ and $\lim_{x \to 1^-} h(x) = 2$

   For h to be continuous at $x = 1 \longrightarrow c + d = 2$.

   <u>Test for continuity at $x = 2$</u>

   i. $h(2) = 4c + d$

   ii. $\lim_{x \to 2^+} h(x) = 8$ and $\lim_{x \to 2^-} h(x) = 4c + d$

   For h to be continuous at $x = 2 \longrightarrow 4c + d = 8$.

   Solving $c + d = 2$ and $4c + d = 8$ simultaneously yields $c = 2$ and $d = 0$

   $\therefore$ for h to be continuous on $(-\infty, \infty)$, $c = 2$ and $d = 0$.

10. Since $f(-2) \nexists$ f is discontinuous at $x = -2$.

    Since $\lim_{x \to -2} f(x) = -6$ the limit exists and the discontinuity is removable.

    We redefine f as $f(x) = \begin{cases} \dfrac{x^2 - 2x - 8}{x - 2} & x \neq -2 \\ -6 & x = -2 \end{cases}$.

11. Since $f(-4) \nexists$ f is discontinuous at $x = -4$.

    Since $\lim_{x \to -4} f(x) = 48$ the limit exists and the discontinuity is removable.

    We redefine f as $f(x) = \begin{cases} \dfrac{x^3 + 64}{x + 4} & x \neq -4 \\ 48 & x = -4 \end{cases}$.

12. Since $f(9) \nexists$ f is discontinuous at $x = 9$.

    Since $\lim_{x \to 9} f(x) = \dfrac{1}{6}$ the limit exists and the discontinuity is removable.

    We redefine f as $f(x) = \begin{cases} \dfrac{3 - \sqrt{x}}{9 - x} & x \neq 9 \\ \dfrac{1}{6} & x = 9 \end{cases}$.

13. <u>Test for continuity at $x = -2$</u>

    i. $f(-2) = 4$

    ii. $\lim_{x \to -2^+} f(x) = 4$ and $\lim_{x \to -2^-} f(x) = 4 \longrightarrow \lim_{x \to -2} f(x) = 4$

    Since $f(-2) = \lim_{x \to -2} f(x)$, f is continuous at $x = -2$

    <u>Test for continuity at $x = 2$</u>

    i. $f(2) = 0$

    ii. $\lim_{x \to 2^+} f(x) = 3$ and $\lim_{x \to 2^-} f(x) = 0 \longrightarrow \lim_{x \to 2} f(x) \nexists$

    Since $\lim_{x \to 2} f(x) \nexists$, f is discontinuous at $x = 2$

14. <u>Test for continuity at $x = -3$</u>

   i. $f(-3) = 0$

   ii. $\lim_{x \to -3^+} f(x) = 0$ and $\lim_{x \to -3^-} f(x) = 0 \longrightarrow \lim_{x \to -2} f(x) = 0$

   Since $f(-3) = \lim_{x \to -3} f(x)$, f is continuous at $x = -3$

   <u>Test for continuity at $x = 3$</u>

   i. $f(3) = 11$

   ii. $\lim_{x \to 3^+} f(x) = 11$ and $\lim_{x \to 3^-} f(x) = 0 \longrightarrow \lim_{x \to 3} f(x) \nexists$

   Since $\lim_{x \to 3} f(x) \nexists$, f is discontinuous at $x = 3$

15. $10 = c^3 - c^2 + c \longrightarrow c \approx 2.365$
16. $-1 = c^5 - 2c^3 + c^2 + 2 \longrightarrow c \approx -1.764$
17. Since $f(0) = 1 > 0$ and $f(1) = -1 < 0 \longrightarrow f(x) = 0$ for some $x \in (0, 1)$.
18. Since $f(0) = -1 < 0$ and $f(1) = 1 > 0 \longrightarrow f(x) = 0$ for some $x \in (0, 1)$.