1. $f'(x) = -\dfrac{1}{x^2}$

   <u>Critical Numbers</u>
   $f' \exists \forall\, x \in [1, 2]$
   $f'(x) \neq 0$
   $\therefore$ no critical numbers

   <u>Extreme Value Theorem</u>
   $f(1) = 1$ (max)
   $f(2) = \dfrac{1}{2}$ (min)

   Since $1(2-1) = 1$ and $\dfrac{1}{2}(2-1) = \dfrac{1}{2} \longrightarrow \displaystyle\int_1^2 \frac{1}{x}\,\mathrm{d}x \in \left[\frac{1}{2}, 1\right]$

2. $f'(x) = \dfrac{3x^2}{2\sqrt{x^3+1}}$

   <u>Critical Numbers</u>
   $f' \exists \forall\, x \in [0, 2]$
   $f'(x) = 0$ when $x = 0$

   <u>Extreme Value Theorem</u>
   $f(0) = 1$ (min)
   $f(2) = 3$ (max)

   Since $1(2-0) = 2$ and $3(2-0) = 6 \longrightarrow \displaystyle\int_0^2 \sqrt{x^3+1}\,\mathrm{d}x \in [2, 6]$

3. $f'(x) = \dfrac{2x^3}{\sqrt{1+x^4}}$

   <u>Critical Numbers</u>
   $f' \exists \forall\, x \in [-1, 1]$
   $f'(x) = 0$ when $x = 0$

   <u>Extreme Value Theorem</u>
   $f(-1) = \sqrt{2}$ (max)
   $f(1) = \sqrt{2}$
   $f(0) = 1$ (min)

   Since $\sqrt{2}(1 - -1) = 2\sqrt{2}$ and $1(1 - -1) = 2 \longrightarrow \displaystyle\int_{-1}^1 \sqrt{1+x^4}\,\mathrm{d}x \in \left[2, 2\sqrt{2}\right]$

4. $f_{\text{avg}} = \dfrac{1}{3}\displaystyle\int_0^3 (x^2 - 2x)\,\mathrm{d}x = \frac{1}{3}\left(\frac{1}{3}x^3 - x^2\right)\Bigg|_0^3 = 0$

5. $f_{\text{avg}} = \dfrac{1}{1 - -1}\displaystyle\int_{-1}^1 x^4\,\mathrm{d}x = \frac{1}{2}\left(\frac{1}{5}x^5\right)\Bigg|_{-1}^1 = \frac{1}{5}$

6. $f_{\text{avg}} = \dfrac{1}{\dfrac{\pi}{4} - -\dfrac{\pi}{2}} \displaystyle\int_{-\pi/2}^{\pi/4} \sin^2 x \cos x \, \mathrm{d}x \approx .191$

7. $f_{\text{avg}} = \dfrac{1}{4-1} \displaystyle\int_{1}^{4} \dfrac{1}{x} \, \mathrm{d}x = \dfrac{1}{3} \ln |x| \Big|_{1}^{4} = \dfrac{2 \ln 2}{3}$

8. $f_{\text{avg}} = \dfrac{1}{2-0} \displaystyle\int_{0}^{2} (4 - x^2) \, \mathrm{d}x = \dfrac{1}{2} \left(4x - \dfrac{1}{3} x^3 \right) \Big|_{0}^{2} = \dfrac{8}{3}$

   Now,
   $4 - c^2 = \dfrac{8}{3} \longrightarrow c = \dfrac{2\sqrt{3}}{3}$ or $c = -\dfrac{2\sqrt{3}}{3} \therefore c = \dfrac{2\sqrt{3}}{3}$

9. $f_{\text{avg}} = \dfrac{1}{3-0} \displaystyle\int_{0}^{3} (4x - x^2) \, \mathrm{d}x = \dfrac{1}{3} \left(2x^2 - \dfrac{1}{3} x^3 \right) \Big|_{0}^{3} = 3$

   Now,
   $4c - c^2 = 3 \longrightarrow c = 1$ or $c = 3 \therefore c = 1$ or $c = 3$ (both are in $[0, 3]$.)

10. $f_{\text{avg}} = \dfrac{1}{2-0} \displaystyle\int_{0}^{2} (x^3 - x + 1) \, \mathrm{d}x = \dfrac{1}{2} \left(\dfrac{1}{4} x^4 - \dfrac{1}{2} x^2 + x \right) \Big|_{0}^{2} = 2$

    Now,
    $c^3 - c + 1 = 2 \longrightarrow c \approx 1.325 \therefore c \approx 1.325$

11. $f_{\text{avg}} = \dfrac{1}{\sqrt{\pi}} \displaystyle\int_{0}^{\sqrt{\pi}} x \sin x^2 \, \mathrm{d}x = \dfrac{1}{\sqrt{\pi}}$

    Now,
    $c \sin c^2 = \dfrac{1}{\sqrt{\pi}} \longrightarrow c \approx .851$ or $c = 1.673 \therefore c \approx .851$ or $c = 1.673$