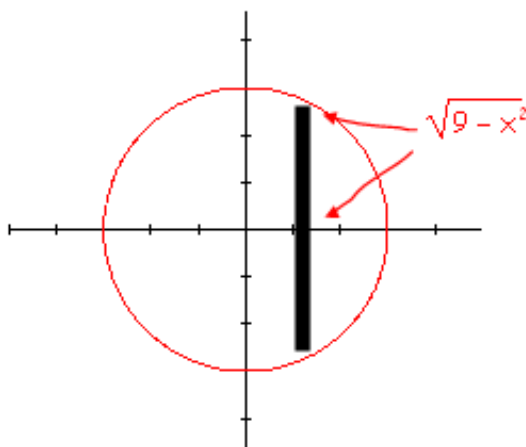


**Note:** On many (most) of these solutions only the setup and answer is given. You should *always show any and all mathematics* you do including showing where intersections come from, integrations, etc. You should also *always* include an appropriately labeled diagram.

1. Side of square:  $2\sqrt{9 - x^2}$

Area of square cross-section:  $(2\sqrt{9 - x^2})^2 = 4(9 - x^2)$



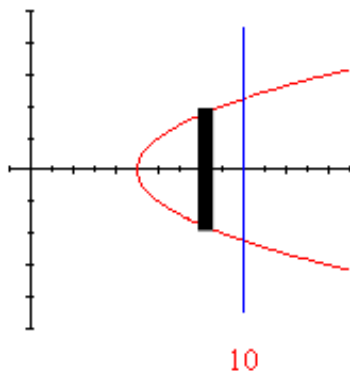
$$V = \int_{-3}^3 4(9 - x^2) dx = 144$$

2. Area of equilateral triangle:  $\frac{\sqrt{3}}{4}(\text{base})^2$

Length of side of equilateral triangle:  $2\sqrt{x - 5}$

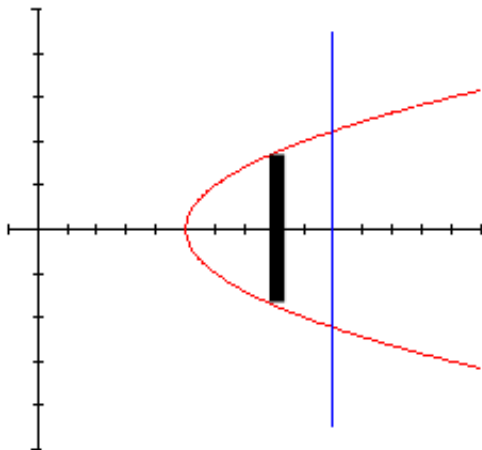
Area of equilateral cross-section:  $\frac{\sqrt{3}}{4} (2\sqrt{x - 5})^2 = \sqrt{3}(x - 5)$

Note: The following diagram is not completely labeled!



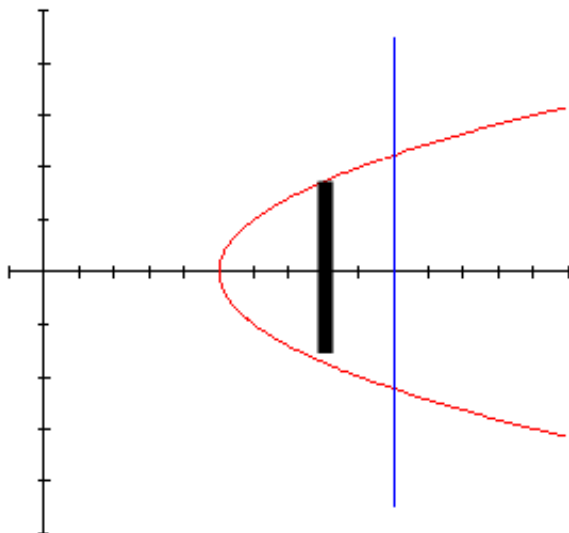
$$V = \int_5^{10} \sqrt{3}(x - 5) dx = \frac{25\sqrt{3}}{2}$$

3. From  $x$ -axis up to curve:  $\sqrt{x-5}$   
 Length of base of square:  $2\sqrt{x-5}$   
 Area of square cross-section:  $(2\sqrt{x-5})^2 = 4(x-5)$   
 Note: The following diagram is not completely labeled!



$$V = \int_5^{10} 4(x-5) dx = 50$$

4. Radius of semicircle:  $\sqrt{x-5}$   
 Area of semicircle:  $\frac{1}{2}\pi r^2$   
 Area of semicircle cross-section:  $\frac{1}{2}(\sqrt{x-5})^2 = \frac{\pi}{2}(x-5)$   
 Note: The following diagram is not completely labeled!



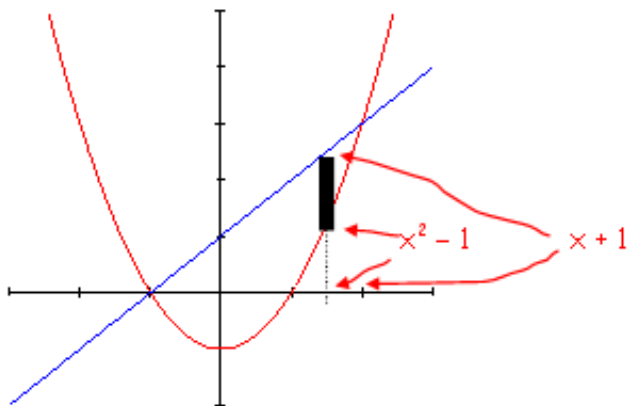
$$V = \frac{\pi}{2} \int_5^{10} (x-5) dx = \frac{25\pi}{4}$$

5. Intersections

$$x + 1 = x^2 - 1 \longrightarrow x = -1 \text{ or } x = 2$$

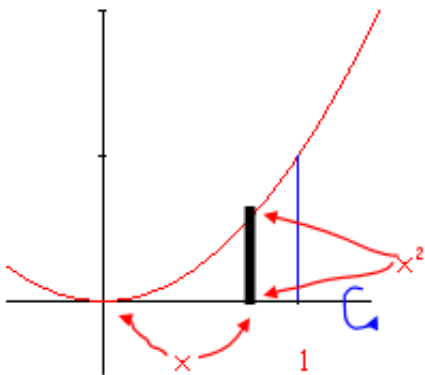
Base of square:  $(x + 1) - (x^2 - 1)$

Area of square cross-section:  $[(x + 1) - (x^2 - 1)]^2$



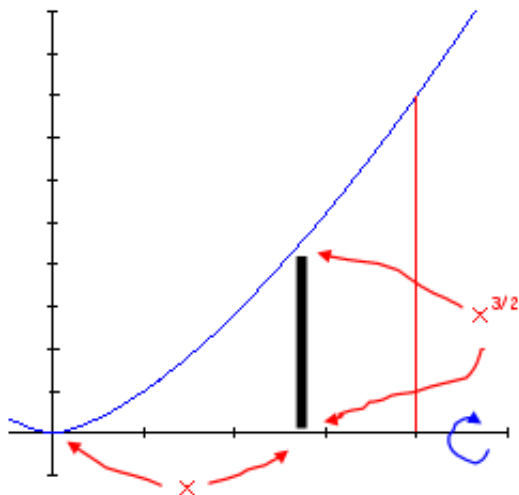
$$V = \int_{-1}^2 [(x + 1) - (x^2 - 1)]^2 dx = \frac{81}{10}$$

6. Radius of disk:  $x^2$



$$V = \pi \int_0^1 (x^2)^2 dx = \frac{\pi}{5} x^5 \Big|_0^1 = \frac{\pi}{5}$$

7. Radius of disk:  $x^{3/2}$



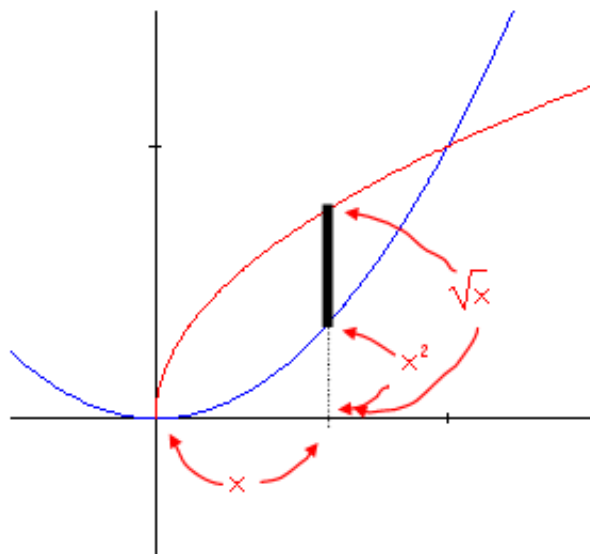
$$V = \pi \int_0^4 \left(x^{3/2}\right)^2 dx = \frac{\pi}{4} x^4 \Big|_0^4 = 64\pi$$

8. Intersections

$$x^2 = \sqrt{x} \rightarrow x = 0 \text{ or } x = 1$$

Outer radius of washer:  $\sqrt{x}$

Inner radius of washer:  $x^2$



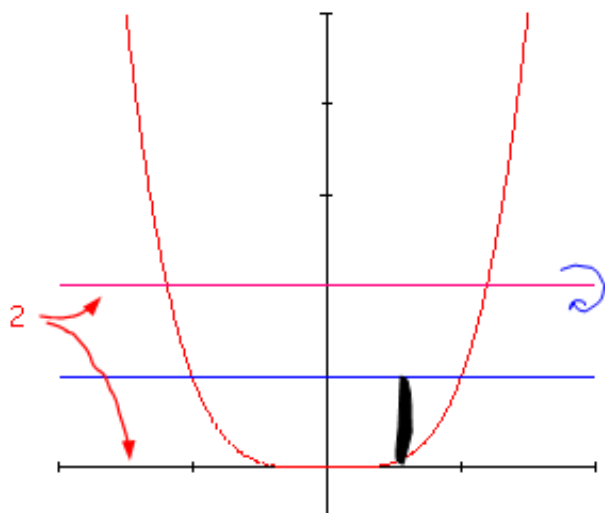
$$V = \pi \int_0^1 \left[ (\sqrt{x})^2 - (x^2)^2 \right] dx = \pi \int_0^1 (x - x^4) dx = \frac{3\pi}{10}$$

9. Intersections

$$x^4 = 1 \longrightarrow x = -1 \text{ or } x = 1$$

Outer radius of washer:  $2 - x^4$

Inner radius of washer: 1

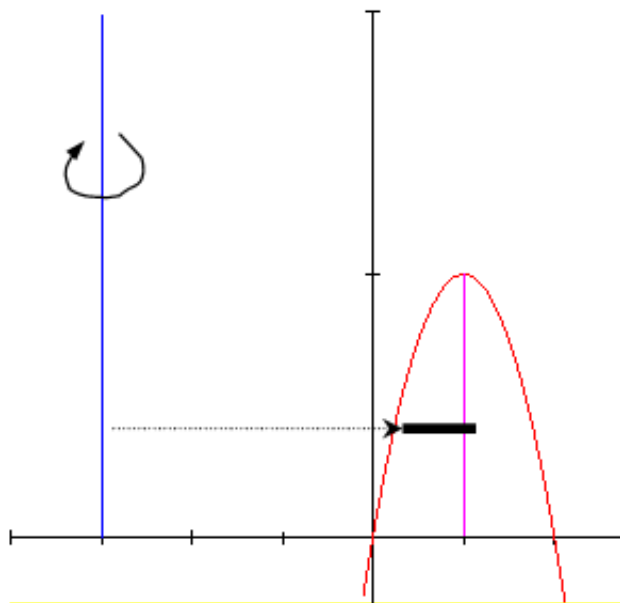


$$V = \pi \int_{-1}^1 \left[ (2 - x^4)^2 - 1^2 \right] dx = \frac{208\pi}{45}$$

10. This problem needs to be done in term of  $y$ .

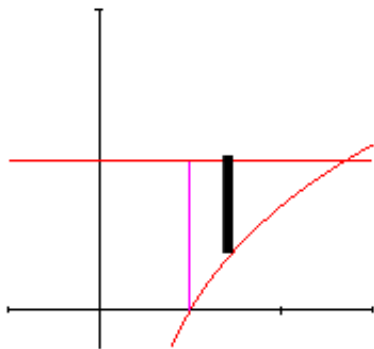
Outer radius of washer: 4

Inner radius of washer:  $\left[ 3 + (1 - \sqrt{1 - y}) \right]^2$



$$V = \pi \int_0^1 \left[ (4)^2 - \left[ 3 + (1 - \sqrt{1 - y}) \right]^2 \right] dy = \frac{29\pi}{6}$$

11. Again, the sketch below is not labeled, yours should be. You should also show how you determined any intersections.

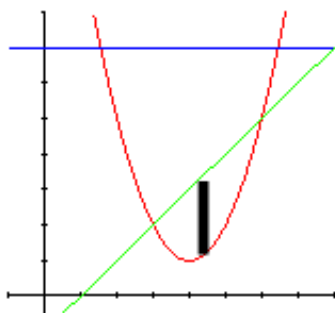


Outer radius of washer: 1

Inner radius of washer:  $\ln x$

$$V = \pi \int_1^e [(1)^2 - (\ln x)^2] dx$$

12. Again, the sketch below is not labeled, yours should be. You should also show how you determined any intersections.

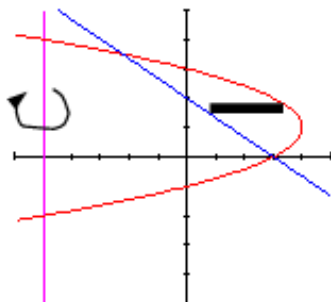


Outer radius of washer:  $7 - [(x - 4)^2 + 1]$

Inner radius of washer:  $7 - (x - 1)$

$$V = \pi \int_3^6 \left[ [7 - [(x - 4)^2 + 1]]^2 - [7 - (x - 1)]^2 \right] dx$$

13. Again, the sketch below is not labeled, yours should be. You should also show how you determined any intersections.



Outer radius of washer:  $5 + (4 - (y - 1)^2)$

Inner radius of washer:  $5 + \frac{6-3y}{2}$

$$V = \pi \int_0^{7/2} \left[ \left[ 5 + (4 - (y - 1)^2) \right]^2 - \left[ 5 + \frac{6 - 3y}{2} \right]^2 \right] dy$$

14. Outer radius of washer: 2  
 Inner radius of washer:  $x^{1/3}$

$$V = \pi \int_0^8 \left[ (2)^2 - (x^{1/3})^2 \right] dx = \frac{64\pi}{5}$$

15. Radius of disk:  $2 - x^{1/3}$

$$V = \pi \int_0^8 \left( 2 - x^{1/3} \right)^2 dx = \frac{16\pi}{5}$$

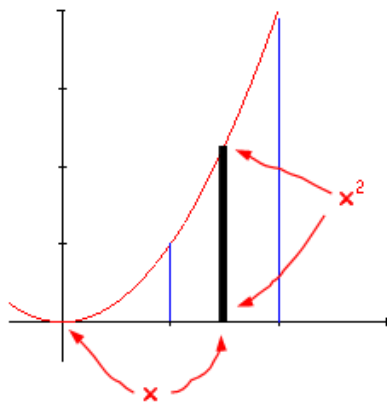
16. Outer radius of washer:  $8 - y^3$   
 Inner radius of washer:  $8 - 4y$

$$V = \pi \int_0^2 \left[ (8 - y^3)^2 - (8 - 4y)^2 \right] dy = \frac{832\pi}{21}$$

17. Outer radius of washer:  $2 - \frac{1}{4}x$   
 Inner radius of washer:  $2 - x^{1/3}$

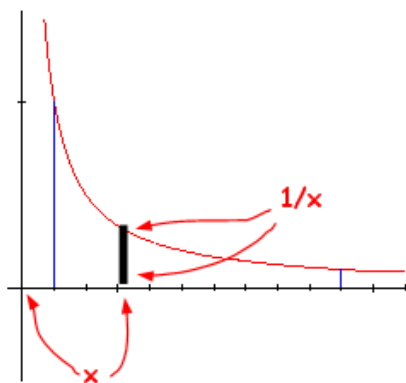
$$V = \pi \int_0^8 \left[ \left( 2 - \frac{1}{4}x \right)^2 - \left( 2 - x^{1/3} \right)^2 \right] dx = \frac{112\pi}{15}$$

18. Radius of shell:  $x$   
 Height of shell:  $x^2$



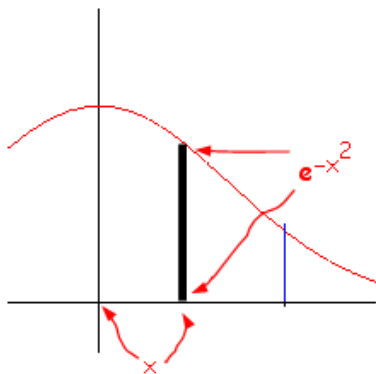
$$V = 2\pi \int_1^2 (x)(x^2) dx = \frac{15\pi}{2}$$

19. Radius of shell:  $x$   
 Height of shell:  $\frac{1}{x}$



$$V = 2\pi \int_1^{10} (x) \left( \frac{1}{x} \right) dx = 18\pi$$

20. Radius of shell:  $x$   
 Height of shell:  $e^{-x^2}$



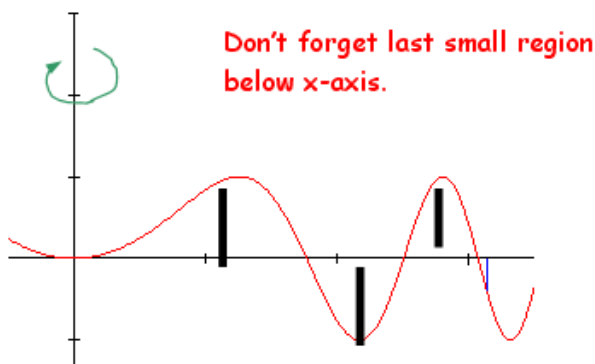
$$V = 2\pi \int_0^1 (x) \left( e^{-x^2} \right) dx = \frac{(e-1)\pi}{e} \approx 1.986$$



21. You actually have four regions. The radius of each region is  $x$  and the height of each shell is either  $x \sin x^2$  or  $-x \sin x^2$ .

Roots

$$\sin x^2 = 0 \rightarrow x = 1.772 \text{ or } x = 3.070 \text{ or } x = 2.507$$



$$V = 2\pi \int_0^{\pi} |x \sin x^2| dx$$

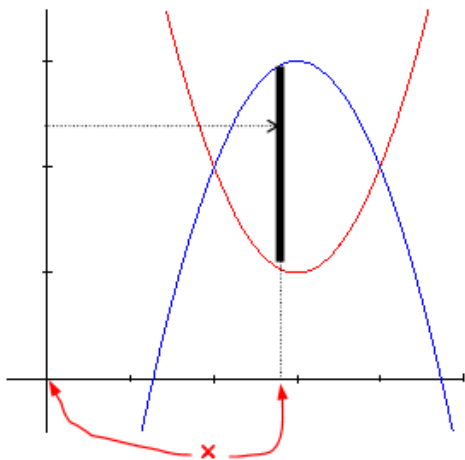
$$V = 2\pi \int_0^{1.772} x \sin x^2 dx - 2\pi \int_{1.772}^{2.507} x \sin x^2 dx + 2\pi \int_{2.507}^{3.070} x \sin x^2 dx - 2\pi \int_{3.070}^{\pi} x \sin x^2 dx \approx 19.155$$

22. Intersections

$$x^2 - 6x + 10 = -x^2 + 6x - 6 \rightarrow x = 2 \text{ or } x = 4$$

Radius of shell:  $x$

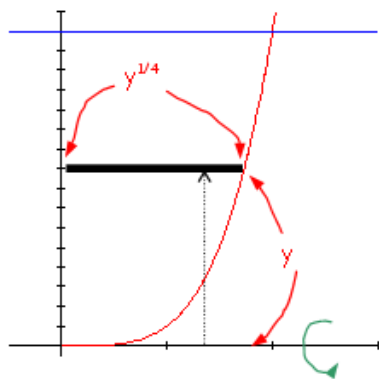
Height of shell:  $(-x^2 + 6x - 6) - (x^2 - 6x + 10)$



$$V = 2\pi \int_2^4 x [(-x^2 + 6x - 6) - (x^2 - 6x + 10)] dx = 16\pi$$

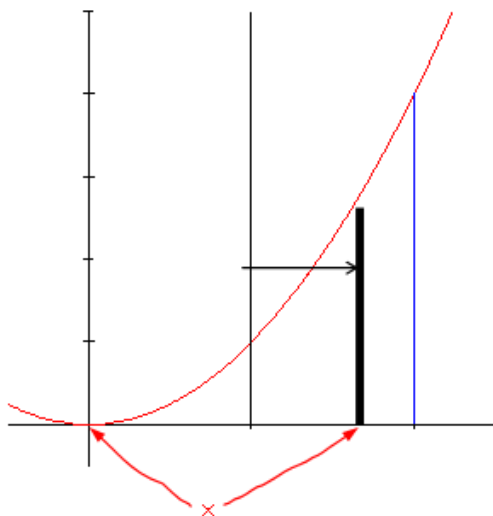
23. This problem must be done in terms of  $y$ .

Radius of shell:  $y$   
 Height of shell:  $y^{1/4}$



$$V = 2\pi \int_0^{16} y (y^{1/4}) dy = \frac{4096\pi}{9}$$

24. Radius of shell:  $x - 1$   
 Height of shell:  $x^2$



$$V = 2\pi \int_1^2 (x - 1)x^2 dx = \frac{17\pi}{6}$$

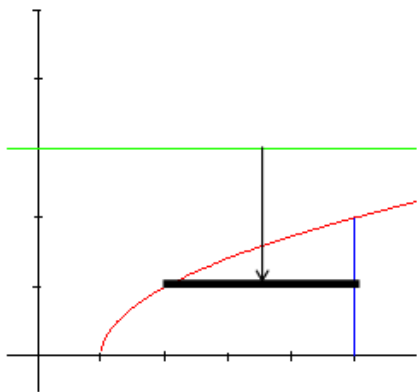
25. This problem must be done in terms of  $y$ .

$$y = \sqrt{x-1} \rightarrow x = y^2 + 1$$

The coordinates of the point where the left end of the element touches the curve can be written  $(y^2 + 1, y)$  and the intersection of the curve and  $x = 5$  is  $(5, 2)$ .

Radius of shell:  $5 - (y^2 + 1)$

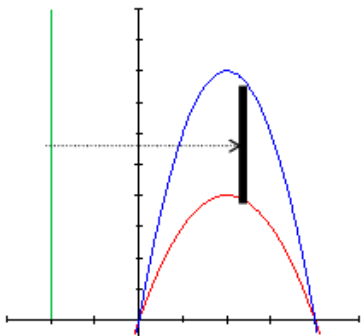
Height of shell:  $3 - y$



$$v = 2\pi \int_0^2 (3 - y) [5 - (y^2 + 1)] dy = 24\pi$$

26. Radius of shell:  $2 + x$

Height of shell:  $(8x - 2x^2) - (4x - x^2)$



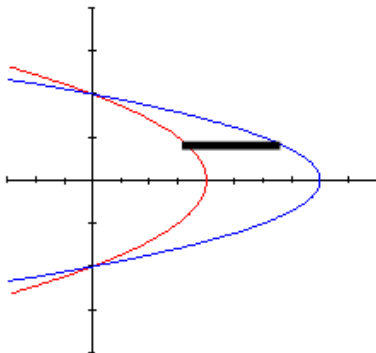
$$V = 2\pi \int_0^4 (2 + x) [(8x - 2x^2) - (4x - x^2)] dx = \frac{256\pi}{3}$$

27. Intersections

$$4 - y^2 = 8 - 2y^2 \longrightarrow y = -2 \text{ or } y = 2$$

Radius of shell:  $5 - y$

Height of shell:  $(8 - 2y^2) - (4 - y^2)$



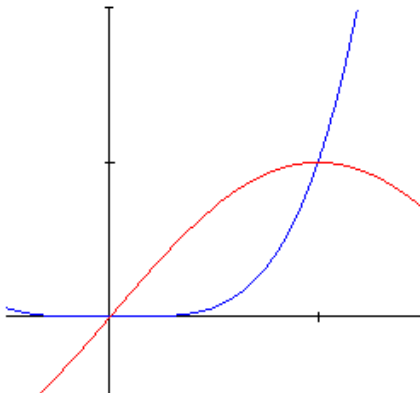
$$V = 2\pi \int_{-2}^2 (5 - y) [(8 - 2y^2) - (4 - y^2)] dy$$

28. Intersections

$$x^4 = \sin \frac{\pi x}{2} \longrightarrow x = 0 \text{ or } x = 1$$

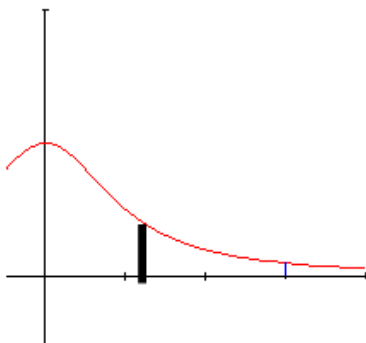
Radius of shell:  $1 + x$

Height of shell:  $\sin \frac{\pi x}{2} - x^4$



$$v = 2\pi \int_0^1 (1 + x) \left[ \sin \frac{\pi x}{2} - x^4 \right] dx$$

29. Radius of shell:  $x$   
 Height of shell:  $\frac{1}{1+x^2}$



$$V = 2\pi \int_0^3 (x) \frac{1}{1+x^2} dx$$