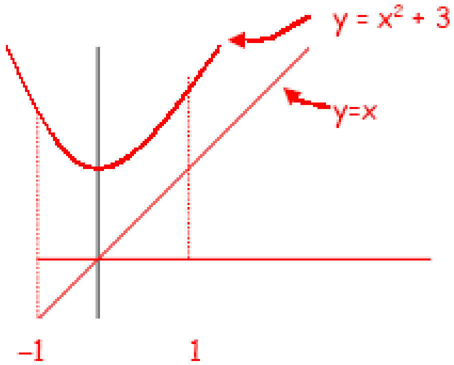
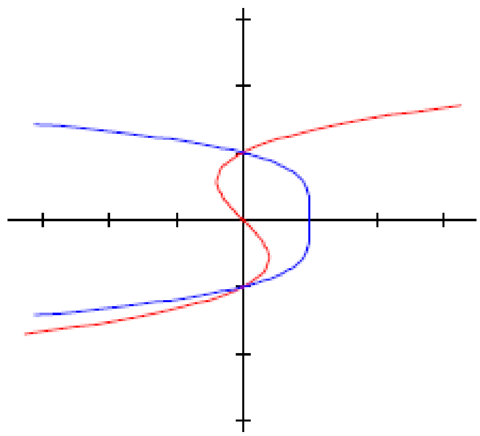
1. Sketch.



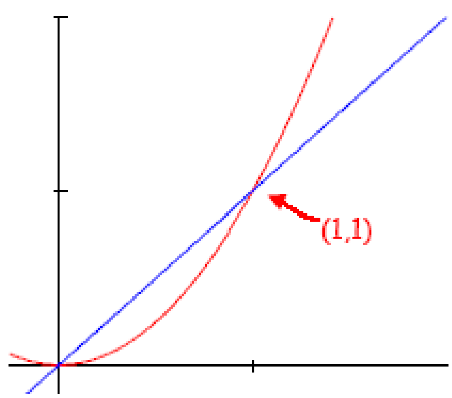
$$
\begin{aligned}
A &= \int_{-1}^{1} [(x^2+3) - x] \, \mathrm{d}x \\
&= \int_{-1}^{1} (x^2 - x + 3) \, \mathrm{d}x \\
&= \left(\frac{1}{3}x^3 - \frac{1}{2}x^2 + 3x \right) \Big|_{-1}^{1} \\
&= \left(\frac{1}{3}(1)^3 - \frac{1}{2}(1)^2 + 3(1) \right) - \left(\frac{1}{3}(-1)^3 - \frac{1}{2}(-1)^2 + 3(-1) \right) \\
&= \frac{20}{3}
\end{aligned}
$$

2. <u>Intersections</u>
$y^3 - y = 1 - y^4 \longrightarrow y = -1$ or $y = 1$



$$
\begin{aligned}
A &= \int_{-1}^{1} \left[(1 - y^4) - (y^3 - y)\right] \mathrm{d}y \\
&= \left(y - \frac{1}{5}y^5 - \frac{1}{4}y^4 + \frac{1}{2}y^2\right)\Big|_{-1}^{1} \\
&= \left(1 - \frac{1}{5} - \frac{1}{4} + \frac{1}{2}\right) - \left(-1 + \frac{1}{5} - \frac{1}{4} + \frac{1}{2}\right) \\
&= \frac{8}{5}
\end{aligned}
$$

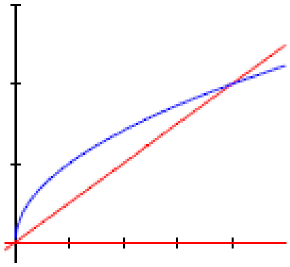
3. <u>Intersections</u>
$x^2 = x \longrightarrow x^2 - x = 0 \longrightarrow x = 0$ or $x = 1$



$$
\begin{aligned}
A &= \int_{0}^{1} (x - x^2)\, \mathrm{d}x \\
&= \left(\frac{1}{2}x^2 - \frac{1}{3}x^3\right)\Big|_{0}^{1} \\
&= \left(\frac{1}{2} - \frac{1}{3}\right) - 0 \\
&= \frac{1}{6}
\end{aligned}
$$

4. Intersections

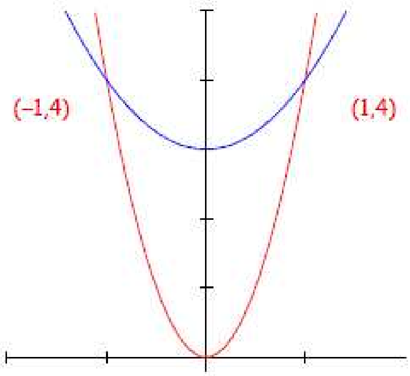
$\sqrt{x} = \frac{1}{2}x \longrightarrow x = 0$ or $x = 4$



$$\begin{aligned} A &= \int_0^4 \left(\sqrt{x} - \frac{x}{2}\right) \mathrm{d}x \\ &= \left(\frac{2}{3}x^{3/2} - \frac{1}{4}x^2\right)\Big|_0^4 \\ &= \frac{4}{3} \end{aligned}$$

5. Intersections

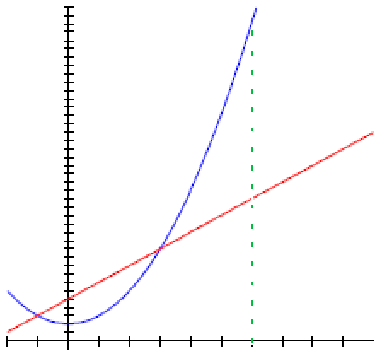
$4x^2 = x^2 + 3 \longrightarrow x = -1$ or $x = 1$



$$\begin{aligned} A &= \int_{-1}^1 \left[(x^2+3) - 4x^2\right] \mathrm{d}x \\ &= \int_{-1}^1 \left(3 - 3x^2\right) \mathrm{d}x \\ &= (3x - x^3)\Big|_{-1}^1 \\ &= (3-1) - (-3+1) \\ &= 4 \end{aligned}$$

6. Intersections

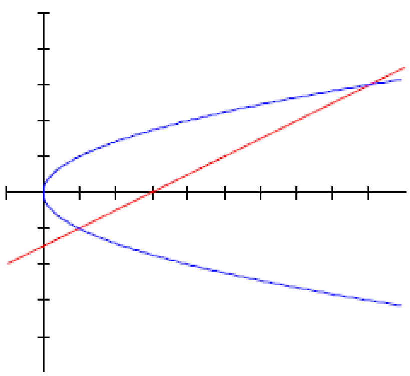
$x^2 + 2 = 2x + 5 \longrightarrow x = -1$ or $x = 3$



$$\begin{aligned} A &= \int_0^3 \left[(2x+5) - (x^2+2)\right] \mathrm{d}x + \int_3^6 \left[(x^2+2) - (2x+5)\right] \mathrm{d}x \\ &= \int_0^3 \left(-x^2+2x+3\right) \mathrm{d}x + \int_3^6 \left(x^2-2x-3\right) \mathrm{d}x \\ &= \left(-\frac{1}{3}x^3 + x^2 + 3x\right)\Big|_0^3 + \left(\frac{1}{3}x^3 - x^2 - 3x\right)\Big|_3^6 \\ &= [(-9+9+9) - 0] + (72-36-18) - (9-9-9) \\ &= 36 \end{aligned}$$

7. Intersections

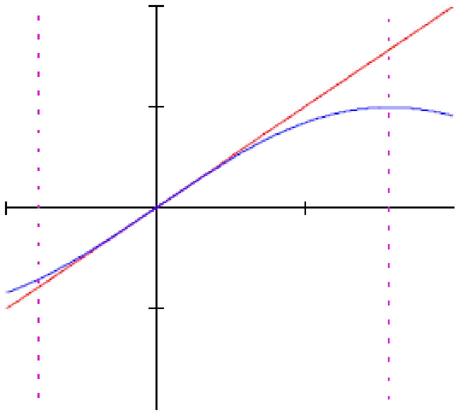
$y^2 = 2y + 3 \longrightarrow y = -1$ or $y = 3$



$$\begin{aligned} A &= \int_{-1}^3 \left[(2y+3) - y^2\right] \mathrm{d}y \\ &= \left(-\frac{1}{3}y^3 + y^2 + 3y\right)\Big|_{-1}^3 \\ &= (-9+9+9) - \left(\frac{1}{3} + 1 - 3\right) \\ &= \frac{32}{3} \end{aligned}$$

8. Intersections

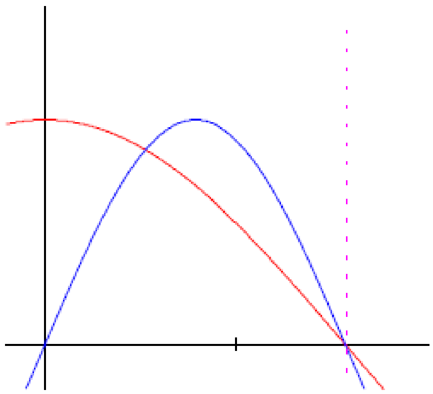
$\sin x = x$ on $\left[-\frac{\pi}{4}, \frac{\pi}{2}\right] \longrightarrow x = 0$



$$
\begin{aligned}
A &= \int_{-\pi/4}^{0} ((\sin x) - x)\,\mathrm{d}x + \int_{0}^{\pi/2} (x - \sin x)\,\mathrm{d}x \\
&= \left(-\cos x - \frac{1}{2}x^2\right)\Big|_{-\pi/4}^{0} + \left(\frac{1}{2}x^2 + \cos x\right)\Big|_{0}^{\pi/2} \\
&= \left[(-1-0) - \left(-\frac{\sqrt{2}}{2} - \frac{\pi^2}{32}\right)\right] + \left[\left(\frac{\pi^2}{8} + 0\right) - (0+1)\right] \\
&= .249
\end{aligned}
$$

9. Intersections

$\cos x = \sin 2x$ on $\left[0, \frac{\pi}{2}\right] \longrightarrow x = \frac{\pi}{2}$ or $\frac{\pi}{6}$



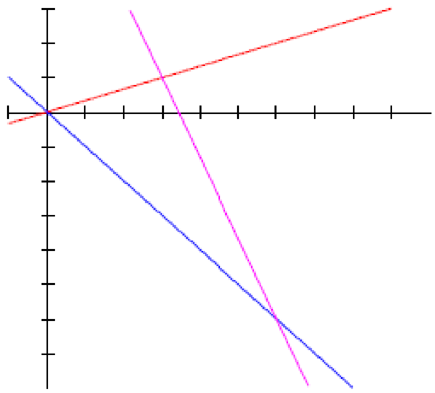
$$
\begin{aligned}
A &= \int_0^{\pi/6} (\cos x - \sin 2x)\,\mathrm{d}x + \int_{\pi/6}^{\pi/2} (\sin 2x - \cos x)\,\mathrm{d}x \\
&= \left[\sin x + \frac{1}{2}\cos 2x\right]\Bigg|_0^{\pi/6} + \left[-\frac{1}{2}\cos 2x - \sin x\right]\Bigg|_{\pi/6}^{\pi/2} \\
&= \left[\left(\frac{1}{2} + \frac{1}{4}\right) - \left(0 + \frac{1}{2}\right)\right] + \left[\left(\frac{1}{2} - 1\right) - \left(-\frac{1}{4} - \frac{1}{2}\right)\right] \\
&= \frac{1}{2}
\end{aligned}
$$

10. Intersections

$-x = \frac{1}{3}x \longrightarrow x = 0$

$\frac{1}{3}x = -\frac{7}{3}x + 8 \longrightarrow x = 3$

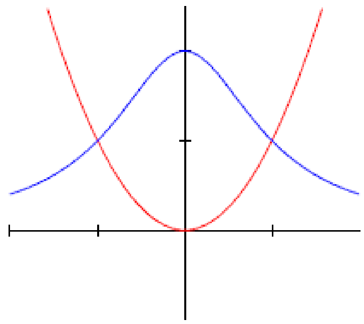
$-\frac{7}{3}x + 8 = -x \longrightarrow x = 6$



$$\begin{aligned}
A &= \int_0^3 \left[\frac{1}{3}x - (-x)\right] \mathrm{d}x + \int_3^6 \left[-\frac{7}{3}x + 8 - (-x)\right] \mathrm{d}x \\
&= \left(\frac{1}{6}x^2 + \frac{1}{2}x\right)\Big|_0^3 + \left(-\frac{7}{6}x^2 + 8x + \frac{1}{2}x^2\right)\Big|_3^6 \\
&= \left[\left(\frac{3}{2} + \frac{9}{2}\right) - 0\right] + \left[(-42 + 48 + 18) - \left(-\frac{21}{2} + 24 + \frac{9}{2}\right)\right] \\
&= 12
\end{aligned}$$

11. Intersections

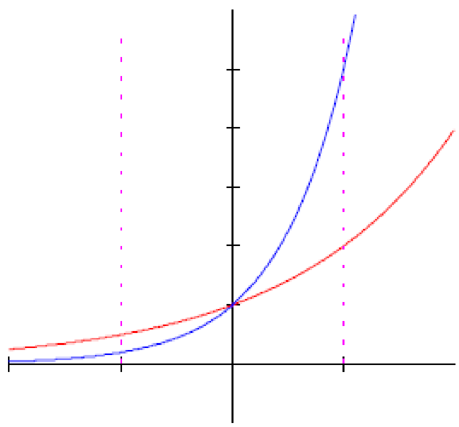
$$x^2 = \frac{2}{x^2+1} \longrightarrow x = 1 \text{ or } x = -1$$



$$\begin{aligned} A &= \int_{-1}^{1} \left[\frac{2}{x^2+1} - x^2\right] \mathrm{d}x \\ &= \left(2\tan^{-1} x - \frac{1}{3}x^3\right)\Big|_{-1}^{1} \\ &= \left((2)\frac{\pi}{4} - \frac{1}{3}\right) - \left((2)\frac{-\pi}{4} + \frac{1}{3}\right) \\ &= \pi - \frac{2}{3} \end{aligned}$$

12. Intersections

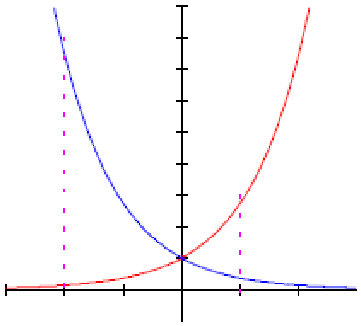
Both exponential functions pass through $(0, 1)$



$$\begin{aligned} A &= \int_{-1}^{0} (2^x - 5^x)\,\mathrm{d}x + \int_{0}^{1} (5^x - 2^x)\,\mathrm{d}x \\ &= \left(\frac{2^x}{\ln 2} - \frac{5^x}{\ln 5}\right)\Big|_{-1}^{0} + \left(\frac{5^x}{\ln 5} - \frac{2^x}{\ln 2}\right)\Big|_{0}^{1} \\ &= \left[\left(\frac{1}{\ln 2} - \frac{1}{\ln 5}\right) - \left(\frac{1}{2\ln 2} - \frac{1}{5\ln 5}\right)\right] + \left[\left(\frac{5}{\ln 5} - \frac{2}{\ln 2}\right) - \left(\frac{1}{\ln 5} - \frac{1}{\ln 2}\right)\right] \\ &\approx 1.267 \end{aligned}$$

13. Intersections

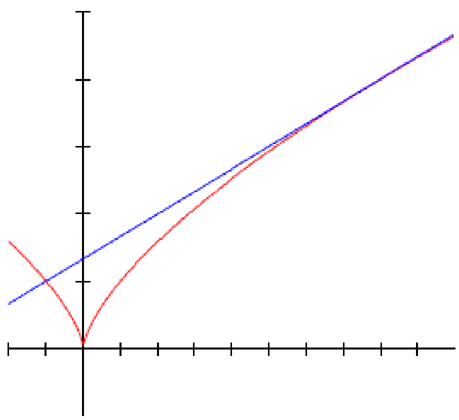
Both exponential functions pass through $(0, 1)$



$$\begin{aligned} A &= \int_{-2}^{0} \left(e^{-x} - e^{x}\right) \mathrm{d}x + \int_{0}^{1} \left(e^{x} - e^{-x}\right) \mathrm{d}x \\ &= \left(-e^{-x} - e^{x}\right)\Big|_{-2}^{0} + \left(e^{x} + e^{-x}\right)\Big|_{0}^{1} \\ &= \left[(-1-1) - \left(-e^{2} - \frac{1}{e^{2}}\right)\right] + \left[\left(e + \frac{1}{e}\right) - (1+1)\right] \\ &= \frac{e^{4} + e^{3} - 4e^{2} + e + 1}{e^{2}} \\ &\approx 6.611 \end{aligned}$$

14. Intersections

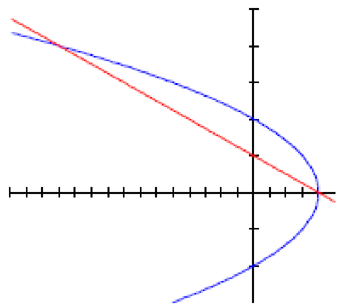
$x^{2/3} = \frac{1}{3}x + \frac{4}{3} \longrightarrow x = -1$ or $x = 8$



$$\begin{aligned} A &= \int_{-1}^{8} \left[\left(\frac{1}{3}x + \frac{4}{3}\right) - x^{2/3}\right] \mathrm{d}x \\ &= \left(\frac{1}{6}x^{2} + \frac{4}{3}x - \frac{3}{5}x^{5/3}\right)\Big|_{-1}^{8} \\ &= \left(\frac{64}{6} + \frac{32}{3} - \frac{96}{5}\right) - \left(\frac{1}{6} + \frac{4}{3} - \frac{3}{5}\right) \\ &= \frac{27}{10} \end{aligned}$$

15. Intersections

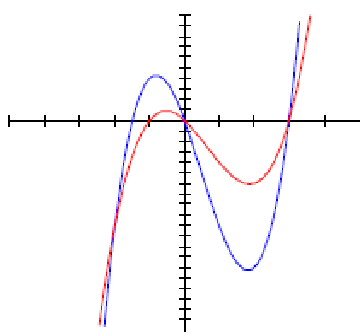
$4 - y^2 = 4 - 4y \longrightarrow y = 0$ or $y = 4$



$$\begin{aligned} A &= \int_0^4 \left[(4 - y^2) - (4 - 4y)\right] \mathrm{d}y \\ &= \int_0^4 (4y - y^2)\,\mathrm{d}y \\ &= \left(2y^2 - \frac{1}{3}y^3\right)\Big|_0^4 \\ &= \left(8 - \frac{64}{3}\right) - 0 \\ &= \frac{32}{3} \end{aligned}$$

16. Intersections

$x^3 - 2x^2 - 3x = 2x^3 - 3x^2 - 9x \longrightarrow x = -2$ or $x = 0$ or $x = 3$



$$\begin{aligned} A &= \int_{-2}^0 \left[\left(2x^3 - 3x^2 - 9x\right) - \left(x^3 - 2x^2 - 3x\right)\right] \mathrm{d}x + \int_0^3 \left[\left(x^3 - 2x^2 - 3x\right) - \left(2x^3 - 3x^2 - 9x\right)\right] \mathrm{d}x \\ &= \frac{253}{12} \end{aligned}$$