1. 18
2. π
3. $5 - \sqrt{5}$
4. $2 - x$
5. $|x+1| = \begin{cases} x+1 & x \geq -1 \\ -x-1 & x < -1 \end{cases}$
6. $x^2 + 1$ since $x^2 + 1 \geq 0 \, \forall \, x$
7. $|2x-3| = \begin{cases} 2x-3 & x \geq 3/2 \\ 3-2x & x < 3/2 \end{cases}$
8. This inequality describes all the x's that are within 5 units of 2.
9. This inequality describes all the x's that are at least 3 units from 3.
10. This inequality describes all the x's that are within 6 units of -3.
11. $4x < 2x + 1$ and $2x + 1 < 3x + 2$
$\therefore \; x \in \left(-1, \dfrac{1}{2}\right)$
12. Change the problem to $x - 6 < 3 - 2x \leq 1 - x$
$x - 6 \leq 3 - 2x$ and $3 - 2x < 1 - x$
$\therefore \; x \in [2, 3)$

When solving inequalities, you can use sign charts or use what you know about lines, parabolas and cubics as explained in class.

13. $(x-2)(x-1) > 0$
   - $(x-2)(x-1) = 0$ when $x = 2$ or $x = 1$
   - $(x-2)(x-1) \, \exists \, \forall \, x$

   Since $(x-2)(x-1) > 0$ on $(-\infty, 1) \cup (2, \infty) \;\longrightarrow\; x \in (-\infty, 1) \cup (2, \infty)$.
14. $2x^2 + x - 1 \leq 0$
   - $2x^2 + x - 1 = 0$ when $x = -1$ or $x = \dfrac{1}{2}$
   - $2x^2 + x - 1 \, \exists \, \forall \, x$

   Since $2x^2 + x - 1 \leq 0$ on $x \in \left[-1, \dfrac{1}{2}\right] \;\longrightarrow\; x \in \left[-1, \dfrac{1}{2}\right]$.
15. $x^2 + x + 1 > 0$
   - $x^2 + x + 1 \neq 0$
   - $x^2 + x + 1 \, \exists \, \forall \, x$

   By completing the square we know that the graph of x^2+x+1 is an upward opening parabola with vertex at $(-1/2, 3/4)$, thus $x^2 + x + 1 > 0 \, \forall \, x \; \therefore \; x \in (-\infty, \infty)$.

16. $x^2 \leq 3 \;\longrightarrow\; x^2 - 3 \leq 0$

- $x^2 - 3 = 0$ when $x = \sqrt{3}$ or $x = -\sqrt{3}$
- $x^2 - 3 \,\exists\, \forall\, x$

You can use a sign chart, as always, but it is generally quicker to look at a quick sketch.
Since $x^2 - 3 \leq 0$ on $\left[-\sqrt{3}, \sqrt{3}\right] \;\longrightarrow\; x^2 \leq 3$ when $x \in \left[-\sqrt{3}, \sqrt{3}\right]$.

17. $x^3 - x^2 \leq 0$

- $x^3 - x^2 \;\exists\; \forall\; x$
- $x^3 - x^2 = 0$ when $x^2(x-1) = 0 \;\longrightarrow\; x = 0$ or $x = 1$.

(Note: A quick sketch will show a positive cubic that touches the x-axis at $x = 0$ and goes above the x-axis at $x = 1$.)

Since $x^3 - x^2 \leq 0$ on $(-\infty, 1] \;\longrightarrow\; x \in (-\infty, 1]$

18. $x^3 > x \;\longrightarrow\; x^3 - x > 0$

- $x^3 - x \,\exists\, \forall\, x$
- $x^3 - x = 0$ when $x = 0$ or $x = 1$ or $x = -1$

Since $x^3 - x > 0$ on $(-1, 0) \cup (1, \infty) \;\longrightarrow\; x^3 > x$ when $x \in (-1, 0) \cup (1, \infty)$.

19. $\dfrac{1}{x} < 4 \;\longrightarrow\; \dfrac{1-4x}{x} < 0$

- $\dfrac{1-4x}{x} \nexists$ when $x = 0$
- $\dfrac{1-4x}{x} = 0$ when $x = \dfrac{1}{4}$

Since $\dfrac{1-4x}{x} < 0$ on $(-\infty, 0) \cup \left(\dfrac{1}{4}, \infty\right) \;\longrightarrow\; \dfrac{1}{x} < 4$ when $x \in (-\infty, 0) \cup \left(\dfrac{1}{4}, \infty\right)$.

20. $\dfrac{4}{x} < x \;\longrightarrow\; \dfrac{4-x^2}{x} < 0$

- $\dfrac{4-x^2}{x} \nexists$ when $x = 0$
- $\dfrac{4-x^2}{x} = 0$ when $x = 2$ or $x = -2$

Since $\dfrac{4-x^2}{x} < 0$ on $(-2, 0) \cup (2, \infty) \;\longrightarrow\; \dfrac{4}{x} < x$ when $x \in (-2, 0) \cup (2, \infty)$.

21. $\dfrac{2x+1}{x-5} < 3 \;\longrightarrow\; \dfrac{16-x}{x-5} < 0$

- $\dfrac{16-x}{x-5} \nexists$ when $x = 5$
- $\dfrac{16-x}{x-5} = 0$ when $x = 16$

Since $\dfrac{16-x}{x-5} < 0$ on $(-\infty, 5) \cup (16, \infty) \;\longrightarrow\; \dfrac{2x+1}{x-5} < 3$ when $x \in (-\infty, 5) \cup (16, \infty)$.

22. $\dfrac{x^2-1}{x^2+1} \geq 0$ (This problem is best done with a sign chart.)

- $\dfrac{x^2-1}{x^2+1}$ $\exists \forall\ x$
- $\dfrac{x^2-1}{x^2+1} = 0$ when $x = 1$ or $x = -1$

$\dfrac{x^2-1}{x^2+1} \geq 0$ when $x \in (-\infty, -1] \cup [1, \infty)$.

23. $|2x| = 3$

$2x = 3$ or $2x = -3$

$x = \dfrac{3}{2}$ $\qquad$ $x = -\dfrac{3}{2}$

24. $|x+3| = |2x+1| \longrightarrow |x+3| = 2x+1$

$x+3 = 2x+1$ or $x+3 = -2x-3$

$x = 2$ $\qquad$ $x = -\dfrac{4}{3}$

25. $|x| < 3$

$x < 3$ and $x > -3$

$\therefore\ x \in (-3, 3)$

26. $|x-4| < 1$

$x - 4 < 1$ and $x - 4 > -1$

$x < 5$ $\qquad$ $x > 3$

$\therefore\ x \in (3, 5)$

27. $|x-5| \geq 2$

$x - 5 \geq 2$ or $x - 5 \leq -2$

$x \geq 7$ $\qquad$ $x \leq 3$

$\therefore\ x \in (-\infty, 3] \cup [7, \infty)$

28. $|5x-2| < 6$

$5x - 2 < 6$ and $5x - 2 > -6$

$x < \dfrac{8}{5}$ $\qquad$ $x > -\dfrac{4}{5}$

$\therefore\ x \in \left(-\dfrac{4}{5}, \dfrac{8}{5}\right)$

29. $\left|\dfrac{x}{2+x}\right| < 1$

$\dfrac{x}{2+x} < 1$ and $\dfrac{x}{2+x} > -1$

$\dfrac{-2}{x+2} < 0$ $\qquad$ $\dfrac{2x+2}{x+2} > 0$

Both of these are non-linear inequalities and are best dealt with via sign charts.

$x \in (-2, \infty)$ and $x \in (-\infty, -2) \cup (-1, \infty)$

$\therefore\ x \in (-1, \infty)$

30. $\left|\dfrac{2-3x}{1+2x}\right| \leq 4$

$$\frac{2-3x}{1+2x} \leq 4 \qquad \text{and} \qquad \frac{2-3x}{1+2x} \geq -4$$

$$\frac{-2-11x}{2x+1} \leq 0 \qquad\qquad \frac{5x+6}{2x+1} \geq 0$$

Both of these are non-linear inequalities and are best dealt with via sign charts.

$x \in (-\infty, -1/2) \cup [-2/11, \infty)$ and $x \in (-\infty, -6/5] \cup (-1/2, \infty)$

$$\therefore\ x \in (-\infty, -6/5] \cup [-2/11, \infty)$$